Asymptotic Safety of Gravity with Matter

含物质的引力的渐近安全

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Abstract

摘要

The asymptotic-safety paradigm posits that the symmetry of quantum theories of gravity and matter is enhanced to quantum scale symmetry, i.e., scale symmetry in the presence of quantum fluctuations, at very high energies. To achieve such a symmetry enhancement, the effect of quantum fluctuations must balance out. It is to be expected that such a balance can only be achieved within a set of theories with limited field content and interaction structure. In this chapter, we review how much is known about these limits.

渐近安全范式假定，在极高能下，引力与物质量子理论的对称性会增强为量子标度对称性，即存在量子涨落时的标度对称性。要实现这种对称性增强，量子涨落的效应必须相互平衡。可以预见，这种平衡只能在一组场内容与相互作用结构受限的理论中实现。在本章中，我们综述了关于这些限制的现有研究成果。

From the quantum scale-invariant regime, the theory transits to a theory with distinct physical scales - most importantly masses for various elementary particles - at low energies. There, quantum scale invariance can leave its imprint in relations between various interactions and mass scales of the theory. These relations can be compared to experimental data, which has two possible implications: first, if the relations do not match the data, the underlying quantum theory of gravity and matter, formulated at and beyond the Planck scale, has been ruled out using experimental data from energies much below the Planck scale. Second, if the relations match the data, the asymptotic-safety paradigm provides a first-principles derivation of free parameters of the Standard Model. Most importantly, this may include the ratios of the Higgs mass to the electroweak scale as well as the value of the fine-structure constant. Similarly, theories beyond the Standard Model may come with fewer free parameters than in their effective-field-theory incarnation without gravity. This may lead to an explanation of the smallness of neutrinos masses and predictions for the nature and interactions of dark matter.

从量子标度不变区，理论在低能下会转变为一个拥有不同物理标度的理论——最重要的就是各类基本粒子的质量。在低能区，量子标度不变性会在理论的不同相互作用和质量标度之间的关系中留下印记。这些关系可以和实验数据对比，这会带来两种可能的结果: 第一，如果关系和数据不匹配，这个建立在普朗克能标及以上的引力与物质基础量子理论，就可以用远低于普朗克能标的实验数据排除；第二，如果关系和数据匹配，渐近安全范式就能从第一性原理推导出标准模型的自由参数，最重要的是，这可能包括希格斯质量与电弱标度的比值，以及精细结构常数的值。类似地，超出标准模型的理论，和不引入引力的有效场论形式相比，自由参数会更少。这还可以解释中微子质量为什么很小，还能对暗物质的性质和相互作用给出预言。

Keywords

关键词

Asymptotic safety - Quantum gravity and matter - Standard Model - Scale invariance - Beyond Standard Model

渐近安全-量子引力与物质-标准模型-标度不变性-超出标准模型

Invitation: Matter Matters in Quantum Gravity

引言: 物质研究在量子引力中至关重要

Motivation: Why Matter Matters

研究动机: 为何物质不可忽略

There are two reasons why matter matters in quantum gravity (by matter, we mean all nongravitational degrees of freedom, including the scalar, fermionic, and gauge fields of the Standard Model as well as potential beyond-Standard-Model fields): the first is theoretical and relates to the differences between theories of quantum gravity only versus theories of all fundamental interactions and matter; the second is phenomenological and relates to observational tests of quantum gravity. We expand on both reasons below.

量子引力研究中，物质不可忽略有两大原因 (此处的物质指所有非引力自由度，包括标准模型的标量场、费米子场、规范场，以及可能存在的超出标准模型的新场): 第一个原因是理论层面，关系到仅量子引力理论与囊括所有基本相互作用和物质的理论之间的差异；第二个是唯象层面，关系到量子引力的观测检验。我们下文将对两个原因展开说明。

The theoretical search for a quantum theory of gravity is often conducted in a setting without matter. The underlying rationale says that a viable quantum theory of pure gravity can be constructed first, and matter added later, in such a way that key properties of the pure-gravity theory remain intact. The rationale would break down in two cases (in principle, there is a third scenario, where the pure gravitational theory is not UV complete, and matter degrees of freedom induce a UV completion):

量子引力的理论研究通常在无物质的框架下开展。背后的逻辑是: 可以先构建一个自洽的纯引力量子理论，之后再添加物质，且纯引力理论的核心性质不会因此改变。这一逻辑在两种情况下会不成立 (原则上还存在第三种场景: 纯引力理论本身不紫外完备，是物质自由度诱导了紫外完备):

First, it would break down if key features of matter-gravity theories would be very different from those of pure-gravity theories, for instance, if the quantum theory cannot be ultraviolet (UV) complete with matter. An analogy for this case is non-Abelian Yang-Mills theory with and without matter. Without matter, the quantum theory is asymptotically free. With matter, asymptotic freedom can be lost in the quantum theory, changing the very nature of the ultraviolet completion.

第一种情况: 如果物质-引力理论的核心特征与纯引力理论差异极大，例如量子理论加入物质后就无法实现紫外 (UV) 完备，该逻辑就不成立。对此可以用有无物质的非阿贝尔杨-米尔斯理论来类比: 无物质时，量子理论是渐近自由的；加入物质后，量子理论会失去渐近自由，彻底改变紫外完备的性质。

Second, it would break down if the coupling to quantum gravity would not render the matter sector UV complete, because the combined theory would then not be UV complete. In many quantum-gravity approaches, it is argued that UV completeness of the matter sector is not an issue, because there is a fundamental Planck-scale cutoff due to fundamental spacetime discreteness. This is actually insufficient for a properly UV complete theory, because such a theory should not just be free of divergences (in the sense of Landau poles, not in the sense of loop divergences removable by renormalization) but also predictive. An effective field theory of the matter sector which comes with a Planckian cutoff is only predictive at energies much lower than the cutoff. At energies close to the cutoff, an infinite number of interactions, each parameterized by its own coupling, can exist. Unless the combination with quantum gravity provides a predictivity principle, the combined matter-gravity theory is not a proper UV complete theory. Such a predictivity principle either sets all but finitely many couplings to zero at Planckian scales or provides an infinite number of relations such that a finite number of free parameters remain.

第二种情况: 如果物质与量子引力耦合后仍无法让物质部门实现紫外完备，那么整个耦合理论就不是紫外完备的，该逻辑也就不成立。在很多量子引力研究方案中，学界通常认为物质部门的紫外完备性不成问题，因为基本的时空离散性会带来普朗克尺度的基本截断。但这实际上不足以得到一个合格的紫外完备理论，因为这类理论不仅要不存在发散 (此处指朗道极点发散，而非可通过重整化消除的圈发散)，还需要具备可预言性。带有普朗克截断的物质部门有效场论，仅在能量远低于截断时才具备预言能力；在能量接近截断时，可以存在无穷多种相互作用，每一种都有各自的耦合参数。除非和量子引力结合后能给出一个预言性原理，否则物质-引力耦合理论就不是合格的紫外完备理论。这样的预言性原理要么会在普朗克尺度下将除有限个之外的所有耦合都置零，要么会给出无穷多关系式，最终只保留有限个自由参数。

Observational test of quantum gravity typically relies on the gravitational effect on matter. For instance, potential quantum-gravity effects in the very early universe are typically looked for in matter observables, such as the cosmic microwave background. Further, quantum-gravity effects in particle physics or even tabletop experiments rely on the interplay of quantum gravity with matter. Finally, even tests relying on putative pure-gravity observables, such as gravitational waves, are ultimately only accessible to us in experimental setups that rely on the interplay of matter and spacetime; see [1] for an overview.

量子引力的观测检验通常依赖引力对物质的效应。例如，极早期宇宙中潜在的量子引力效应，一般都是通过宇宙微波背景这类物质可观测量寻找。此外，粒子物理甚至桌面实验中的量子引力效应，也依赖量子引力与物质的相互作用。最后，哪怕是依赖纯引力可观测量 (比如引力波) 的检验，最终也只有通过依赖物质和时空相互作用的实验装置才能被我们观测到；综述参见文献 [1]。

Additionally, many more observational tests become available at low energies, if one explores matter-gravity systems. For pure gravity, the only requirement is that at low energies, it reduces to general relativity, with higher-order corrections to it being sufficiently small. Current observations constrain curvature-squared couplings to be smaller than 10^{60} [2], which indicates that those are currently only very weakly constrained. In terms of free parameters, one is essentially left with the Newton coupling (and the cosmological constant) as potentially predictable quantities from a quantum theory of gravity.

此外，如果研究物质-引力系统，还能在低能区开展更多观测检验。对于纯引力，唯一的要求是它在低能区退化为广义相对论，且对广义相对论的高阶修正足够小。目前观测约束显示，曲率平方耦合小于 10^{60} [2]，说明这些耦合当前仅受到极弱的约束。在自由参数层面，本质上只有牛顿耦合 (加宇宙学常数) 是量子引力理论可能预言的量。

For gravity-matter theories, there is an additional requirement, namely, that the matter sector reduces to the Standard Model (SM), plus potential dark-matter and other beyond Standard Model (BSM) fields. This provides many more and stronger constraints than the pure-gravity setting does. In terms of free parameters, one gains the additional 19 free parameters of the SM as potentially predictable quantities from a quantum theory of gravity with matter.

对于引力-物质理论，还有一项额外要求: 物质部门需要退化为标准模型 (SM)，加上可能存在的暗物质和其他超出标准模型 (BSM) 的场。这相比纯引力框架能给出更多也更强的约束。在自由参数层面，还会额外得到标准模型的 19 个自由参数，成为含物质量子引力理论可以预言的物理量。

Matter Matters in Asymptotically Safe Gravity

物质在渐近安全引力中至关重要

Here we focus on the asymptotic-safety paradigm. Its starting point is the perturbative nonrenormalizability of gravity, which means that gravity loses predictivity at the Planck scale, because the couplings of all possible interactions are free parameters. Asymptotic safety restores predictivity because an additional symmetry, not easily seen in standard perturbation theory (see, however, [3]), holds above the Planck scale: quantum scale symmetry means that all couplings, made dimensionless by division through an appropriate power of a scale, are constant. This is referred to as a fixed point in the renormalization group flow, which describes how the theory changes with respect to an energy scale. As an analogy, one may view the renormalization group flow as the mathematical counterpart of a microscope, with which one can change the resolution scale at which a system is considered. At a fixed point, changes of the resolution scale do not result in changes of the system, i.e., scale symmetry - a form of self-similarity - is achieved.

我们在此聚焦渐近安全范式。它的出发点是引力的微扰不可重整性: 这意味着引力在普朗克尺度失去预测能力，因为所有可能相互作用的耦合都是自由参数。渐近安全恢复了预测能力，因为在普朗克尺度以上存在一个标准微扰论中不易发现的额外对称性 (不过参见文献 [3]): 量子标度对称性意味着，所有经合适的尺度幂次无量纲化后的耦合都是常数。这在描述理论随能标如何变化的重整化群流中被称为不动点。打个比方，可以把重整化群流视作显微镜的数学对应，借助它我们可以改变观测系统的分辨率标度。在不动点处，改变分辨率标度不会改变系统，也就是说实现了标度对称性——一种自相似形式。

Just like any symmetry in a QFT, quantum scale symmetry relates the values of couplings to each other. The special aspect of quantum scale symmetry is that relations continue to hold at lower energy scales/larger distances scales than the Planck scale, where quantum scale symmetry is no longer realized. The reason for these relations is that departure from quantum scale symmetry is only achievable in a QFT, if particular interactions (so-called relevant ones) are present. These relations restore predictivity and make a fundamental QFT of gravity conceivable. Examples of relevant interactions and the resulting relations will be shown in section ”Toward a UV Completion of the Standard Model” below.

就像量子场论中的所有对称性一样，量子标度对称性将各个耦合的取值相互关联起来。量子标度对称性的特殊之处在于，这些关联在低于普朗克尺度/更大距离尺度 (量子标度对称性在此不再实现) 依然成立。这些关联得以存在的原因是，只有当特定相互作用 (即所谓相关相互作用) 存在时，量子场论才会偏离量子标度对称性。这些关联恢复了预测能力，让引力的基本量子场论成为可能。我们会在下文“迈向标准模型的紫外完备”一节介绍相关相互作用和所得关联的实例。

In the asymptotic-safety paradigm, evidence is starting to accumulate for the following scenarios:

在渐近安全范式中，支持以下情景的证据正不断积累:

- An asymptotically safe pure-gravity fixed point can be step by step deformed to an asymptotically safe fixed point in theories which contain matter degrees of freedom, most importantly the SM. We discuss this in detail in section ”Gravitational Fixed Point Under the Impact of (Minimally Coupled) Matter”.

- 渐近安全的纯引力不动点可以逐步退化为包含物质自由度 (最重要的就是标准模型 SM 的自由度) 的理论中的渐近安全不动点。我们将在“(最小耦合) 物质影响下的引力不动点”一节详细讨论。

- Gravitational fluctuations induce new interactions in the matter sector. All induced interactions respect the symmetries of the kinetic terms, which includes global symmetries. Hence, the properties of the asymptotically safe fixed point may be in part determined by those global symmetries. Note that in the SM, the global symmetries of the kinetic terms are typically broken by the marginal interactions. The presence of global symmetries may therefore be more relevant for BSM physics, e.g., in a dark sector. We discuss this in detail in section ”Global Symmetries Persist and Have Phenomenological Consequences”.

- 引力涨落会在物质区诱生新的相互作用。所有诱生相互作用都满足动能项的对称性，其中包括整体对称性。因此，渐近安全不动点的性质可以部分由这些整体对称性决定。注意，在标准模型中，动能项的整体对称性通常被边缘相互作用破缺。因此整体对称性的存在对超出标准模型 (BSM) 的物理可能更重要，例如暗区。我们将在“整体对称性得以保留并具有唯象学效应”一节详细讨论。

- Under the impact of gravitational fluctuations, the SM becomes UV complete. The Landau poles that the SM on its own contains are substituted by an asymptotically safe, quantum scale-invariant regime.

- 在引力涨落的影响下，标准模型成为紫外完备理论。标准模型本身自带的朗道极点被渐近安全的量子标度不变区取代。

- In the infrared (IR), some of the free parameters of the SM, i.e., some of its perturbatively renormalizable couplings, become calculable quantities that can be predicted from first principles. The technical reason is that they are irrelevant couplings in the asymptotically safe regime. This provides observational tests of asymptotically safe matter-gravity systems. We discuss this in detail in section ”Toward a UV Completion of the Standard Model”.

- 在红外 (IR)，标准模型的部分自由参数，即部分可微扰重整化的耦合，成为可以从第一性原理出发预测的可计算量。技术层面的原因是，它们是渐近安全区的无关耦合。这为渐近安全物质-引力系统提供了观测检验方法。我们将在“迈向标准模型的紫外完备”一节详细讨论。

- Beyond the SM, not all theories are asymptotically safe. This provides predictions for ongoing and future searches for new physics, including the nature of dark matter. We discuss this in detail in section ”Physics Beyond the Standard Model”.

- 超出标准模型的理论中，并非所有理论都是渐近安全的。这为当前和未来的新物理搜寻 (包括暗物质性质研究) 给出了预言。我们将在“超出标准模型的物理”一节详细讨论。

Interplay of Quantum Gravity and Matter and Structure of This Chapter

量子引力与物质的相互作用及本章结构

This chapter is structured as follows: first, we introduce key concepts of asymptotically safe quantum gravity, as well as the most important methods to explore it. Then, we start from investigating asymptotic safety in a system with few interactions and step by step add interactions and later also fields: first, we review how matter that is noninteracting impacts a gravitational fixed point. Then, we discuss the role of symmetries and how they determine which interactions of matter are unavoidably present. Finally, we add those interactions that need to be present in order to obtain a viable phenomenology, including the SM and some physics beyond the SM.

本章结构安排如下: 首先，我们介绍渐近安全量子引力的核心概念，以及研究该理论最重要的研究方法。随后，我们从研究仅存在少量相互作用的系统中的渐近安全性出发，逐步引入更多相互作用，后续再引入更多场: 首先，我们回顾无相互作用物质对引力不动点的影响。接着，我们讨论对称性的作用，以及对称性如何决定物质的哪些相互作用是必然存在的。最后，我们引入得到可行唯象学必须包含的那些相互作用，包括标准模型 (SM) 和部分超出标准模型的新物理。

It is in fact a nontrivial consequence of the methodology used, namely, the functional renormalization group, that the two sides of the interplay of quantum gravity and matter can be ”factorized” at the level of calculations, at least approximately (these approximations consist in neglecting the effect of nonminimal interactions, as well as the impact of the anomalous dimensions of matter fields on the gravitational couplings and vice versa): within approximations, one can consider first the impact of matter fields on quantum gravity and second the effect of quantum gravity and matter, cf. Fig. 1. These two independent studies are combined in a second step, where the fully coupled system is investigated.

事实上，所用研究方法即函数重整化群的一个不平凡结论是，量子引力与物质相互作用的两个方面可以在计算层面至少近似实现“因子分解”(这些近似包括忽略非最小相互作用的效应，以及物质场反常维数对引力耦合的影响，反之亦然): 在近似框架内，我们可以先研究物质场对量子引力的影响，再研究量子引力与物质的共同效应，参见图 1。这两项独立研究会在第二步合并，届时我们将研究完全耦合的系统。

We keep this introduction as nontechnical and pedagogical as possible and highlight the basic mechanisms behind the results. In several sections, we provide Further reading paragraphs, where we discuss some more technical details or the relation to other results.

我们在本次引言中尽可能保持非技术性与教学性，重点介绍结果背后的基本机制。在多个小节中，我们设置了“拓展阅读”段落，在其中讨论更多技术细节或与其他研究结果的关联。

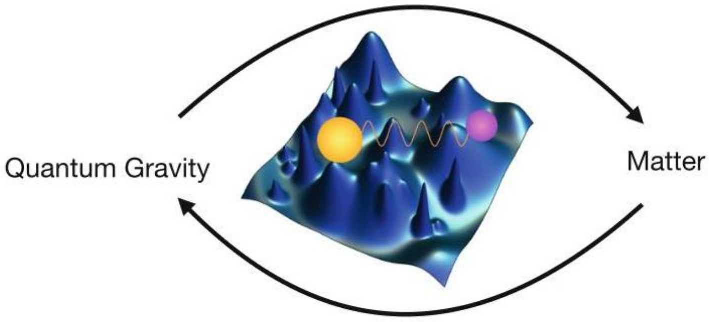


Fig. 1 The interplay of quantum gravity and matter can as a first step be approximately ”factorized” and the effect of matter on the gravitational fixed point be considered separately from the effect of quantum gravity on matter. Therefore, we construct this chapter by considering the impact of matter on quantum gravity first and the effect of quantum gravity on matter second, adding more and more realistic interactions and field content as we go along

图 1 量子引力与物质的相互作用可以在第一步近似实现“因子分解”，物质对引力不动点的影响可以与量子引力对物质的影响分开研究。因此，本章构建方式为: 先研究物质对量子引力的影响，再研究量子引力对物质的影响，在推进过程中逐步引入越来越贴合实际的相互作用与场内容。

Methods to Investigate Asymptotically Safe Gravity-Matter Systems

渐近安全引力-物质系统的研究方法

Since asymptotic safety makes an interacting theory scale invariant, it is characterized by an interacting fixed point of the renormalization group flow, where all scale dependence is lost. A fixed point is called interacting, if interactions are present, i.e., (some of) the couplings are nonzero. Methods to explore such a scale-invariant regime for quantum gravity and matter include the following:

由于渐近安全使一个相互作用理论成为标度不变的，它的特征是重整化群流的相互作用不动点，在该不动点处所有标度依赖性消失。若存在相互作用即 (部分) 耦合不为零，则该不动点称为相互作用不动点。探索量子引力与物质这类标度不变区域的方法包括以下几种:

(i) Perturbative methods

(i) 微扰方法

(ii) Lattice methods

(ii) 格点方法

(iii) Functional methods

(iii) 泛函方法

Due to the interacting nature of the fixed point, couplings are generically nonzero. Thus, perturbative methods, which explore the theory in the vicinity of the noninteracting limit, can only be used to limited extent; see [3,4] for examples.

由于不动点具有相互作用本质，耦合通常不为零。因此，在非相互作用极限附近探究理论的微扰方法适用范围有限；实例参见文献 [3,4]。

However, an interacting theory can often become perturbative, when a parameter in the theory is taken to a particular limit, e.g., a limit of many fields, or of a special spacetime dimensionality; see, e.g.,[5] for examples. In gravity, $d = 2$ is a special dimensionality, because it makes the Newton coupling dimensionless. In the language of statistical physics, we therefore call $d = 2$ the critical dimension d_c for the Newton coupling. It is a general feature that theories that are asymptotically free in their critical dimension d_c are asymptotically safe in $d = d_c + \varepsilon$. Because gravity is also topological in $d = 2$, the situation is somewhat special. However, calculations in $d = 2 + \varepsilon$ show a beta function with a negative quadratic term, i.e., the limit $\varepsilon \to 0$ gives a beta function that exhibits asymptotic freedom. At $\varepsilon > 0$, it therefore admits an asymptotically safe fixed point. Whether or not ε can be extended to $\varepsilon = 2$ is an open question [6-10].

然而，当理论中的某个参数取特定极限时，例如多场极限或特殊时空维数极限，相互作用理论通常可以变为微扰理论；实例参见例如文献 [5]。在引力中，$d = 2$ 是一个特殊维数，因为它使牛顿耦合成为无量纲量。因此在统计物理的语言中，我们将 $d = 2$ 称为牛顿耦合的临界维数 d_c 。在临界维数 d_c 下渐近自由的理论，在 $d = d_c + \varepsilon$ 下渐近安全是一个普遍性质。由于引力在 $d = 2$ 下也是拓扑性的，情况较为特殊。尽管如此，在 $d = 2 + \varepsilon$ 中的计算显示 β 函数带有负二次项，即极限 $\varepsilon \to 0$ 给出的 β 函数表现出渐近自由。因此在 $\varepsilon > 0$ 处，该理论存在一个渐近安全不动点。ε 能否推广到 $\varepsilon = 2$ 仍是一个开放问题 [6-10]。

Lattice methods are powerful tools to discover and characterize scale-invariant regimes. On the lattice, an asymptotically safe fixed point generates a higher-order phase transition in the phase diagram (spanned by the couplings of the system). At the phase transition, the system loses any memory of scales due to diverging correlation lengths, which is characteristic for critical phenomena.

格点方法是发现和表征标度不变区域的有力工具。在格点上，渐近安全不动点会在相图 (由系统的耦合张成) 中产生高阶相变。在相变处，由于关联长度发散，系统失去了所有标度记忆，这是临界现象的特征。

For gravity, one faces the challenge that the lattice itself has to become dynamical, because gravity is not a theory on spacetime but a theory of spacetime, where the path integral sums over different configurations of the lattice. In addition, one may face the challenge that a lattice can (but need not) break spacetime symmetries, e.g., a regular lattice breaks local Lorentz invariance. In dynamical triangulations, the d -dimensional spacetime is discretized in terms of d -dimensional simplices, and the path integral for quantum gravity is expressed as a sum over all possible combinatorics of such triangulations. These are weighted by the exponential of their respective action. In the Euclidean setting of dynamical triangulations, one can run Monte-Carlo simulations of the path integral directly; in the Lorentzian setting of Causal dynamical triangulations, one builds configurations so that they can be Wick-rotated, thereby transforming the Lorentzian path integral to a statistical generating functional that can be explored with Monte-Carlo techniques. Evidence for the existence of a higher-order phase transition, which is necessary to take the continuum limit, has been collected in the case of causal dynamical triangulations [11]; see also [12-15] for recent studies in Euclidean dynamical triangulations. For further discussions, we refer the reader to the section on dynamical triangulations of this handbook.

对于引力，人们面临的挑战是格点本身必须是动力学的，因为引力不是时空上的理论，而是时空本身的理论，其路径积分需要对不同格点构型求和。此外，还可能面临这样的挑战: 格点可以 (但不必然) 破坏时空对称性，例如规则格点会破坏局部洛伦兹不变性。在动力学三角剖分中，d 维时空由 d 维单形离散化，量子引力的路径积分被表示为对所有这类三角剖分的可能组合方式求和。这些组合由各自作用量的指数加权。在动力学三角剖分的欧几里得框架中，可以直接对路径积分进行蒙特卡洛模拟；在因果动力学三角剖分的洛伦兹框架中，人们构造的构型满足维克旋转条件，从而将洛伦兹路径积分转化为可以用蒙特卡洛技术研究的统计生成泛函。对于因果动力学三角剖分，已经收集到了高阶相变存在的证据，而高阶相变是取连续极限的必要条件 [11]；欧几里得动力学三角剖分的最新研究也可参见文献 [12-15]。更多相关讨论请读者参考本手册中关于动力学三角剖分的章节。

In addition, lattice simulations may be based on Regge calculus, which varies not the triangulation but the edge lengths of the building blocks to sum over all spacetime configurations [16]. Further, a combinatorial approach based on random graphs may also feature a second-order phase transition [17, 18], as could another combinatorial approach based on tensor models [19].

此外，格点模拟也可以基于里奇微积分，该方法不改变三角剖分，仅通过改变构造块的边长来对所有时空构型求和 [16]。另外，基于随机图的组合方法也可能存在二级相变 [17, 18]，基于张量模型的另一类组合方法也同样如此 [19]。

These phase transitions need not necessarily constitute the same universality class that is commonly known as ”asymptotically safe gravity,” i.e., while they may be asymptotically safe in a technical sense, their emergent physics may differ from that encoded in the continuum functional approach we discuss below. Note that the terminology ”universality class” is taken from statistical physics/condensed matter, where interacting fixed points have long played an important role, because they characterize continuous phase transitions. A universality class is determined by the dimensionality, field content, and symmetries and quantitatively described by the set of critical exponents, which describe the scaling behavior of physical quantities in the vicinity of the phase transition. The decision whether or not the physics agrees between such different approaches which all search for a second-order phase transition can be based on sufficiently precise calculations of the critical exponents, which uniquely characterize the universality class of a phase transition.

这些相变不一定属于通常所说的“渐近安全引力”同一普适类: 也就是说，尽管它们在技术意义上可能是渐近安全的，但它们的演生物理可能与我们下文讨论的连续统泛函方法所包含的物理不同。请注意，“普适类”这一术语源自统计物理/凝聚态物理，在该领域中相互作用不动点长期发挥着重要作用，因为它们描述了连续相变。普适类由维度、场内容和对称性决定，并由一组临界指数定量描述，临界指数刻画了物理量在相变附近的标度行为。所有这些不同方法都在搜寻二级相变，判断不同方法得到的物理是否一致，可以基于对临界指数的足够精确的计算——临界指数唯一确定了相变的普适类。

Finally, it has been proposed in [20,21] that causal sets, reviewed in another section of this book, may also shed light on asymptotic safety: it is usually assumed that the discreteness scale in causal sets is fixed. However, if one can take it to zero at a higher-order phase transition, one obtains a continuum limit in a genuinely Lorentzian setting.

最后，文献 [20,21] 提出，本书另一节已经评述过的因果集也可以为渐近安全带来启发: 通常假设因果集中的离散标度是固定的。但如果能让它在高阶相变处趋于零，就能在真正的洛伦兹背景下得到连续统极限。

In summary, while current explorations of asymptotically safe gravity are mainly based on functional methods reviewed below, there is certainly scope to extend the toolbox and achieve complementary insights based on other methods or by ”repurposing” other approaches, such as the causal-set approach.

总而言之，尽管目前对渐近安全引力的探索主要基于下文评述的泛函方法，拓展工具盒，借助其他方法或“再利用”包括因果集方法在内的其他方案获得互补性见解，确实仍有空间。

The ideal tool to probe an asymptotically safe theory can do two things: first, it can probe the UV regime to search for scale symmetry. Second, it can connect a scale-symmetric regime in the UV to emergent phenomenology in the IR.

探测渐近安全理论的理想工具需要完成两件事: 第一，它能够探测紫外区以搜寻标度对称性。第二，它能够将紫外的标度对称区和红外的演生冷现象连接起来。

Functional methods are such a tool, because they allow to extract the scale dependence of a system within and beyond perturbation theory. Since most research on asymptotically safe gravity-matter systems relies on functional methods, in particular on the functional renormalization group (FRG), we will briefly introduce the method and some notation in the following.

泛函方法就是这样一种工具，它能够提取微扰论内外系统的标度依赖关系。由于大多数对渐近安全引力-物质系统的研究都依赖泛函方法，尤其是泛函重整化群 (FRG)，我们接下来将简要介绍该方法和部分记号。

The key object in the FRG is the scale-dependent effective action Γ_k . It is a scale-dependent counterpart of the classical action, i.e., it gives rise to the equations of motion for the expectation value of the field. As a function of the RG scale k , it interpolates between the microscopic action $\Gamma_{k\to\infty}$ when no quantum fluctuations are integrated out and the full quantum effective action $\Gamma_{k\to 0}$ when all quantum fluctuations are integrated out. Note that the microscopic action $\Gamma_{k\to\infty}$ is sometimes also referred to as the classical action. This is technically not completely accurate; see [22-24] for the relation between bare (or classical) action and

$\Gamma_{k\to\infty}$. Further, the term "classical action" can be conceptually confusing in the gravitational context: observationally, we know that the action that describes gravity at low curvature scales is S_{EH} , the Einstein-Hilbert action of GR , and we usually refer to it as the classical action, given that GR does not contain quantum effects. However, in the context of asymptotic safety, it is not correct that S_{EH} is the action that is "quantized" in the sense of a path integral $Z = \int \mathcal{D}g_{\mu\nu}e^{iS_{\text{EH}}}$. Instead, S_{EH} should be recovered as the leading approximation to $\Gamma_{k\to 0}$ in the limit of low curvature. We are interested in the scale derivative of Γ_k , i.e., in $k\partial_k\Gamma_k$, because it allows us to do the two things we are interested in: first, finding whether there is a scale-invariant regime, related to $k\partial_k\Gamma_k = 0$, and second, integrating $k\partial_k\Gamma_k$ from the scale-invariant regime in the limit $k \to \infty$ to $k = 0$ to investigate the phenomenology of asymptotic safety.

FRG 的核心对象是依赖标度的有效作用量 Γ_k 。它是经典作用量依赖标度的对应物，即它给出了场期望值的运动方程。作为 RG 标度 k 的函数，它在未积分任何量子涨落时的微观作用量 $\Gamma_{k\to\infty}$ 和积分完所有量子涨落时的全量子有效作用量 $\Gamma_{k\to 0}$ 之间插值。请注意，微观作用量 $\Gamma_{k\to\infty}$ 有时也被称为经典作用量。这在技术上并不完全准确；关于裸 (或经典) 作用量与 $\Gamma_{k\to\infty}$ 的关系参见文献 [22-24]。此外，“经典作用量”这一术语在引力语境下会造成概念混淆: 根据观测，我们知道低曲率标度下描述引力的作用量是 S_{EH} ，即 GR 的爱因斯坦-希尔伯特作用量，我们通常将它称为经典作用量，因为广义相对论不包含量子效应。但在渐近安全的语境下，认为 S_{EH} 是路径积分 $Z = \int \mathcal{D}g_{\mu\nu}e^{iS_{\text{EH}}}$ 意义上被“量子化”的作用量是错误的。相反，S_{EH} 应当是低曲率极限下 $\Gamma_{k\to 0}$ 的领头阶近似结果。我们关注 Γ_k 的标度导数，也就是 $k\partial_k\Gamma_k$ ，因为它能让我们完成我们感兴趣的两件事: 第一，找到是否存在与 $k\partial_k\Gamma_k = 0$ 相关的标度不变区，第二，将 $k\partial_k\Gamma_k$ 从 $k \to \infty$ 极限下的标度不变区积分到 $k = 0$ ，来研究渐近安全的唯象学。

The FRG indeed provides a flow equation for Γ_k , which reads [25-28]

FRG 确实给出了 Γ_k 的流方程，其形式为 [25-28]

$$k\partial_k\Gamma_k = \frac{1}{2}\,\text{sTr}\left[(k\partial_k R_k)\left(\Gamma_k^{(2)} + R_k\right)^{-1}\right]. \tag{1}$$

Here, the right-hand side integrates over quantum fluctuations, with those fluctuations with momenta of the order of k contributing most to the change of Γ_k at k . The technical ingredients of the right-hand side are the second functional derivative of Γ_k with respect to all fields of the system, $\Gamma_k^{(2)}$, the regulator functional R_k , and a supertrace sTr which sums/integrates over all discrete/continuous indices. The combination $\left(\Gamma_k^{(2)} + R_k\right)^{-1}$ is the regularized propagator. In the propagator, R_k acts akin to a scale-dependent mass term, because it appears just like a standard mass term would, together with the momentum, in the schematic form $p^2 + R_k$ or $p^2 + m^2$, respectively. The difference to a standard mass term is that it is not constant, but only present for low-energy modes, $p^2 < k^2$. Therefore, these are suppressed; and a $\sim 1/p^2$ divergence, i.e., an IR divergence, is avoided. Thus, technically speaking, the regulator ensures the IR finiteness of the flow equation. In addition, the physical masses of modes also enter the propagator and ensure that a mode decouples dynamically, once k falls below the mass scale. This is relevant even for massless fluctuations, which couple through a mass-like term: gravity decouples automatically, once $k \simeq M_{\text{Planck}}$; see Fig. 2 for a schematic illustration of the functional RG flow and the different regimes for gravity-matter theories. In the numerator, the scale derivative $k\partial_k R_k$ suppresses quantum fluctuations of high momenta. The two occurrences of R_k therefore realize the Wilsonian idea of integrating out quantum fluctuations according to their momentum in a stepwise fashion. To achieve this, the regulator has to satisfy several conditions, most importantly $R_k\left(p^2\right) > 0$ for $p^2 < k^2$ (where p denotes a four-momentum) to suppress low-energy modes and $R_k\left(p^2\right) = 0$

for $p^2 > k^2$, such that the high-energy modes are also suppressed in the flow equation and only modes with $p^2 \approx k^2$ remain. Since quantum fluctuations are integrated out according to their four-momentum squared, the FRG is best employed in Euclidean settings, where a cutoff on the four-momentum squared indeed distinguishes UV and IR. For steps toward a generalization to Lorentzian spacetimes in the context of quantum gravity, see [29-31].

在此处，右侧对量子涨落做积分，其中动量量级为 k 的涨落对 Γ_k 在 k 处的变化贡献最大。右侧的技术组成部分包括 Γ_k 对系统所有场 $\Gamma_k^{(2)}$ 的二阶泛函导数、调节泛函 R_k ，以及对所有离散/连续指标求和/积分的 supertrace sTr。组合 $\left(\Gamma_k^{(2)} + R_k\right)^{-1}$ 是正规化传播子。在传播子中，R_k 的作用类似于依赖标度的质量项，因为它的出现形式和标准质量项完全一致，和动量共同分别呈现为概型形式 $p^2 + R_k$ 或 $p^2 + m^2$ 。它和标准质量项的区别在于它不是常数，仅存在于低能模式 $p^2 < k^2$ 中。因此这些模式被压低，避免了 $\sim 1/p^2$ 发散，即红外发散。因此从技术上来说，调节器保证了流方程的红外有限性。此外，模式的物理质量也会进入传播子，保证一旦 k 低于质量标度，模式就会发生动力学退耦。这一点甚至对无质量涨落也成立，后者通过类质量项耦合: 一旦 $k \simeq M_{\text{Planck}}$ ，引力就会自动退耦；泛函 RG 流以及引力-物质理论不同区域的概图示见图 2。在分子中，标度导数 $k\partial_k R_k$ 压低了高动量量子涨落。因此两处出现的 R_k 实现了威尔逊的思想，即按照动量分步积出量子涨落。要实现这一点，调节器必须满足若干条件，最重要的是对 $p^2 < k^2$ (其中 p 表示四动量) 满足 $R_k\left(p^2\right) > 0$ 以压低低能模式，对 $p^2 > k^2$ 满足 $R_k\left(p^2\right) = 0$ ，使得高能模式也在流方程中被压低，仅留下动量满足 $p^2 \approx k^2$ 的模式。由于量子涨落是按四动量平方积分的，泛函重整化群 (FRG) 最适合在欧几里得背景下使用，对四动量平方设置截断确实能区分紫外和红外。关于量子引力背景下推广到洛伦兹时空的研究进展，参见 [29-31]。

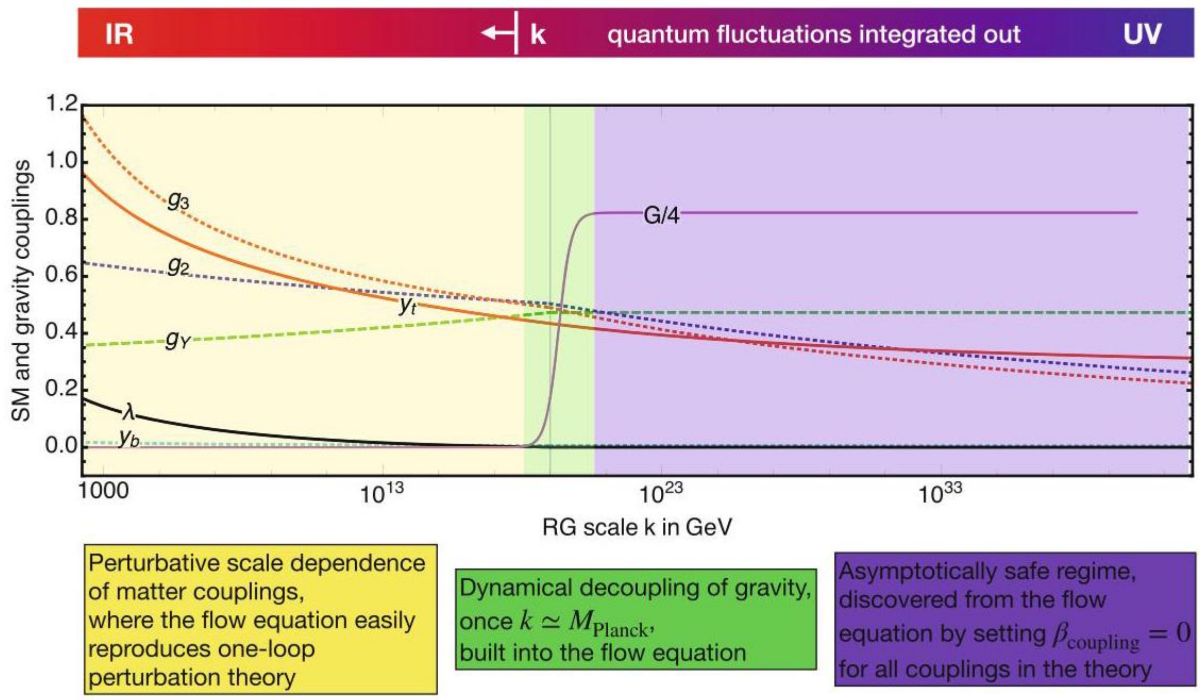


Fig. 2 We illustrate the functional RG flow of gravity coupled to the SM, which has three regimes as a function of k : in the UV, asymptotic safety is realized and discoverable through the flow equation by setting $\beta = 0$ for all couplings. At the Planck scale, gravity decouples dynamically. Below the Planck scale, the SM couplings (here shown are the three gauge couplings g_i , the two largest Yukawa couplings y_t and y_b and

the Higgs quartic self-interaction λ) exhibit a perturbative scale dependence, and the flow equation easily reproduces one-loop perturbation theory. (At even lower scales, in the very deep IR, perturbation theory no longer describes QCD; the FRG can also be used in that regime successfully; see, e.g., [32] for reviews)

Fig. 2 我们展示了引力与标准模型耦合的泛函 RG 流，它随 k 变化存在三个区域: 在紫外区域实现了渐近安全，通过对所有耦合设置 $\beta = 0$ ，即可通过流方程对其进行研究。普朗克能标处引力发生动力学退耦。普朗克能标以下，标准模型耦合 (此处展示了三个规范耦合 g_i 、两个最大的汤川耦合 y_t 和 y_b 以及希格斯四次自相互作用 λ) 呈现微扰标度依赖，流方程可以很容易地重现单圈微扰论。(在更低能标处，极深红外区域微扰论不再能描述量子色动力学，FRG 也可以在该区域成功应用，综述参见例如 [32])

Structurally, the flow equation (1) is of one-loop form, but in terms of the full and regularized propagator, such that it is valid beyond perturbation theory. In fact, $\Gamma_k^{(2)}$ is not just the perturbative expression $p^2 + m^2$ (or the appropriate version for fields which are not scalar) but contains higher-order terms, i.e., it is the inverse propagator fully dressed by quantum fluctuations. Despite this difference, calculations share similarities with perturbative loop calculations and, most importantly, are feasible for a wide range of theories, including gravitational ones.

从结构上看，流方程 (1) 具有单圈形式，但它是基于完全正则化传播子构建的，因此在微扰论之外仍然成立。实际上，$\Gamma_k^{(2)}$ 不仅仅是微扰表达式 $p^2 + m^2$ (或适用于非标量场的对应形式)，它还包含高阶项，即它是被量子涨落完全修正的逆传播子。尽管存在这一区别，此类计算与微扰圈计算仍有相似性，且最重要的是，它适用于包括引力理论在内的大范围理论。

The flow equation is successfully employed in various physical scenarios which are governed by an interacting fixed point; see, e.g., [32] for an overview.

流方程已成功应用于由相互作用不动点主导的各类物理场景；综述参见例如文献 [32]。

Quantum fluctuations generate all interactions compatible with the symmetries of a system. Thus, the scale-dependent effective action Γ_k contains all these interactions, and the scale dependence of the corresponding couplings can be extracted by projecting the left- and right-hand sides of the flow equation (1) on the corresponding interaction. In practice, this is done by taking functional derivatives with respect to the fields. In this way, one obtains the beta function for the coupling of an interaction term.

量子涨落会生成所有与系统对称性相容的相互作用。因此，依赖能标的有效作用量 Γ_k 包含所有这些相互作用，对应耦合常数的能标依赖可以通过将流方程 (1) 的左右两边投影到对应相互作用上提取出来。实际操作中，该操作通过对场取泛函导数完成，由此可以得到相互作用项对应耦合的 beta 函数。

In practice, only a subset of interactions can be accounted for. Practical computations therefore have to restrict the set of terms - kinetic terms and interactions - that enter Γ_k to a typically finite subset. This constitutes a truncation in the space of all interactions and introduces a systematic uncertainty in the results obtained within the method. Extending the truncation by adding more and more interactions into the system decreases this systematic uncertainty; see [33] for an example.

实际计算中只能包含一部分相互作用，因此实用计算必须将进入 Γ_k 的项 (动力学项和相互作用项) 限制在一个有限集合内。这相当于在所有相互作用空间中做了截断，会给该方法得到的结果引入系统误差。通过向系统中加入越来越多的相互作用来扩展截断，可以降低这种系统误差；实例参见文献 [33]。

Crucially, completely random choices of interactions do of course not lead to robust results. Instead, reliable truncations are based on physical insight into the nature of the system, for example, regarding the degree of a system's nonper-turbativeness. Systematic expansion schemes can be employed, e.g., a derivative expansion (including all orders in the field, but subsequent orders in derivatives), a vertex expansion (including all orders in derivatives, but subsequent orders in fields), or an expansion based on canonical dimension (including the most relevant interactions first). Calculations in gravity-matter systems are typically based on the last scheme; based on the assumption that the fixed point is near perturbative, i.e., the canonical dimension remains a useful ordering principle. We will get back to this point at the very end of this chapter (see section "On the Near-Perturbative Nature of Gravity-Matter Systems") to review whether this assumption is self-consistent and supported by the results obtained by basing truncations on it.

至关重要的一点是，完全随机选择相互作用当然无法得到可靠的结果。相反，可靠的截断依赖于对系统性质的物理洞察，例如对系统非微扰程度的判断。可以采用系统展开方案，例如导数展开 (包含场的所有阶，但按导数阶逐阶展开)、顶点展开 (包含导数的所有阶，但按场阶逐阶展开)，或是基于正则维度的展开 (先纳入最相关的相互作用)。引力-物质系统的计算通常基于最后一种方案，其基础假设是不动点接近微扰，即正则维度仍然是一个有效的排序原则。我们会在本章的最后回到这一点 (参见“论引力-物质系统的近微扰性质”一节)，检验该假设是否自洽，以及是否得到基于该假设截断所得结果的支持。

In the discussion above, we have referred to the four-momentum of modes, which is of course a notion closely tied to flat spacetime. On a curved background, it can be generalized: just as p^2 are the eigenvalues of the flat-spacetime d' Alembertian, λ_p are the eigenvalues of a suitable curved-spacetime d' Alembertian $\Box$. However, if spacetime itself is fluctuating, the definition of a suitable generalization of momenta is non-trivial. Therefore, when applying the FRG to gravity, one has to specify an auxiliary background metric, with respect to which the momenta of quantum fluctuations can be measured [28]. This is best implemented via the background-field method (see [34] for details) because this method also allows to preserve a background diffeomorphism invariance, which becomes full diffeomorphism invariance when the auxiliary background metric is removed.

在上述讨论中，我们提到了模式的四动量，这当然是一个与平直时空紧密关联的概念。在弯曲背景下，它可以被推广: 正如 p^2 是平直时空达朗贝尔算符的本征值，λ_p 是合适的弯曲时空达朗贝尔算符 $\Box$ 的本征值。但如果时空本身是涨落的，那么对动量做合适推广的定义并非易事。因此，将 FRG 应用于引力时，必须指定一个辅助背景度规，量子涨落的动量可以相对于该度规测量 [28]。这一点最适合通过背景场方法实现 (细节参见文献 [34])，因为该方法还可以保留背景微分同胚不变性，移除辅助背景度规后就可以得到完整的微分同胚不变性。

Therefore, we split the metric into background metric $\bar{g}_{\mu\nu}$ and a fluctuation field $h_{\mu\nu}$, for example, with a linear split (other choices are possible, most popular among them an exponential split):

因此，我们将度规拆分为背景度规 $\bar{g}_{\mu\nu}$ 和涨落场 $h_{\mu\nu}$ ，例如采用线性拆分 (也可以选择其他拆分方式，其中最常用的是指数拆分):

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \tag{2}$$

where the metric fluctuation $h_{\mu\nu}$ is not restricted to be perturbatively small.

其中度规涨落 $h_{\mu\nu}$ 不要求在微扰意义下很小。

We emphasize that the background is merely a technical ingredient and does not even need to be specified in calculations (although in practice, many studies choose to specify it for reasons of technical simplicity). Given the background metric, the regulator can then be introduced as a mass-like term for $h_{\mu\nu}$, while $\bar{g}_{\mu\nu}$ is specified to measure the "momentum" of $h_{\mu\nu}$. Due to this regularization, Γ_k depends on $\bar{g}_{\mu\nu}$ and $h_{\mu\nu}$ independently. To zeroth order in $h_{\mu\nu}$, i.e., $\Gamma_k\,[\bar{g}; h = 0]$ contains the so-called background couplings, which are the physical ones which enter observable quantities. However, due to the regularized propagator in the flow equation, their scale dependence is driven by those couplings of $\Gamma_k\,[\bar{g}; h]$ appearing at higher orders in an expansion in $h_{\mu\nu}$, the so-called fluctuation couplings. In a widespread approximation scheme, the background-field approximation, this difference is neglected when computing the scale dependence of background couplings. We will provide further details in the further reading part of this section and refer the reader to the chapter on the vertex expansion.

我们需要强调，背景仅仅是一个技术要素，在计算中甚至无需对其加以指定 (尽管实际研究中出于技术简化的考虑，很多工作会选择指定背景)。给定背景度规后，就可以为 $h_{\mu\nu}$ 引入类似质量项的调节项，同时指定 $\bar{g}_{\mu\nu}$ 来度量 $h_{\mu\nu}$ 的“动量”。通过这种正则化，Γ_k 独立依赖于 $\bar{g}_{\mu\nu}$ 和 $h_{\mu\nu}$ 。在 $h_{\mu\nu}$ 的零阶近似下，即 $\Gamma_k\,[\bar{g}; h = 0]$ 包含了所谓的背景耦合，这些耦合是进入可观测量的物理耦合。然而，由于流方程中的正则化传播子，背景耦合的标度依赖由 $h_{\mu\nu}$ 展开高阶项中出现的 $\Gamma_k\,[\bar{g}; h]$ 的那些耦合驱动，后者也就是所谓的涨落耦合。在常用的背景场近似这一近似方案中，计算背景耦合的标度依赖时会忽略上述差异。我们会在本节的拓展阅读部分提供更多细节，也请读者参考顶点展开那一章。

Due to the formulation as a QFT of the metric, it is rather straightforward to couple matter degrees of freedom to gravity in asymptotic safety. When investigating asymptotically safe quantum gravity within the FRG framework, this is especially true, since the continuum formulation allows using standard formulations of scalars, gauge fields, and fermions. This is in contrast with other approaches to quantum gravity, where the mere definition of matter fields can be more involved.

由于该框架是将度规表述为量子场论，在渐近安全中把物质自由度耦合到引力上相当直接。这一点在 FRG 框架下研究渐近安全量子引力时尤其明显，因为连续谱表述可以直接使用标量、规范场和费米子的标准 formulations。这与其他量子引力研究方法形成了对比，在其他方法中，仅仅定义物质场都可能更为复杂。

In summary, this provides a flow equation for gravity-matter systems, which allows to first search for scale symmetry and, second, start from a scale-symmetric regime in the UV and integrate out all quantum fluctuations to obtain the resulting effective dynamics $\Gamma_{k\to 0}$, which can be compared to observations.

综上，这就给出了引力-物质系统的流方程: 它首先可以用来寻找标度对称性，其次可以从紫外的标度对称区域出发，积分掉所有量子涨落，得到最终的有效动力学 $\Gamma_{k\to 0}$ ，该结果可以和观测对比。

Further Reading

延伸阅读

Background-field approximation and fluctuation computations

背景场近似与涨落计算

Due to the regulator (and because one has to gauge-fix $h_{\mu\upsilon}$), Γ_k depends on $\bar{g}_{\mu\upsilon}$ and $h_{\mu\upsilon}$ independently, i.e., $\Gamma_k = \Gamma_k[\bar{g}; h]$.$\Gamma_k[\bar{g}; h]$ can be expanded as a series in metric fluctuations $h_{\mu\upsilon}$, with different couplings at each order in $h_{\mu\upsilon}$. The zeroth order in this expansion, i.e., $\Gamma_k[\bar{g}; h=0]$, contains the so-called background couplings, whose scale dependence is driven by the couplings appearing in higher-order terms of the expansion, the so-called fluctuation couplings. It is a widespread approximation to neglect the difference between background and fluctuation couplings, when computing the scale dependence of $\Gamma_k[\bar{g}; h=0]$. We call this the background-field approximation. Computations which go beyond the zeroth order in the expansion in metric fluctuations will be referred to as fluctuation computations; see also [35].

由于正则化项的存在 (且必须对 $h_{\mu\upsilon}$ 做规范固定)，Γ_k 独立依赖于 $\bar{g}_{\mu\upsilon}$ 和 $h_{\mu\upsilon}$ ，也就是说 $\Gamma_k = \Gamma_k[\bar{g}; h]$.$\Gamma_k[\bar{g}; h]$ 可以按度规涨落 $h_{\mu\upsilon}$ 展开为级数，$h_{\mu\upsilon}$ 的每一阶都有不同的耦合。该展开的零阶项，即 $\Gamma_k[\bar{g}; h=0]$ ，包含了所谓的背景耦合，其标度依赖由展开高阶项中的耦合 (即所谓的涨落耦合) 驱动。在计算 $\Gamma_k[\bar{g}; h=0]$ 的标度依赖时，忽略背景耦合与涨落耦合的差异是一种常用近似，我们将其称为背景场近似。度规涨落展开中超出零阶的计算被称为涨落计算；参见文献 [35]。

Nontrivial symmetry identities, the so-called Nielsen, or split Ward identities, restore background independence; see, e.g., [35,36]. These identities encode the difference between correlation functions of the background field and correlation functions of the fluctuation field.

非平凡对称性恒等式 (即所谓的尼尔森恒等式，也称为分裂沃德恒等式) 可以恢复背景无关性；参见例如文献 [35,36]。这些恒等式刻画了背景场关联函数与涨落场关联函数之间的差异。

In the presence of a fluctuation field $h_{\mu\upsilon}$, the scale-dependent effective action can be expanded in terms of a vertex expansion as

当存在涨落场 $h_{\mu\upsilon}$ 时，依赖标度的有效作用量可以按顶点展开为

$$\Gamma_k[\bar{g}, h] = \Gamma_k[\bar{g}, 0] + \sum_{n=1}^{\infty} \left(\frac{\delta^n \Gamma_k[\bar{g}, h]}{\delta h_{\gamma_1\delta_1} \dots \delta h_{\gamma_n\delta_n}} \bigg|_{h=0} \right) h_{\gamma_1\delta_1} \dots h_{\gamma_n\delta_n}. \tag{3}$$

The first term in this expansion depends only on the background metric, and we refer to the couplings in this term as background couplings. These couplings are the physical couplings that eventually enter observable quantities. The second term in Eq. (3) is the sum over n -point vertices, which generally have different scale dependences than the background couplings. We refer to the couplings appearing in these n -point vertices as fluctuation couplings. We can see from Eq. (3) and the flow equation Eq. (1) that the scale dependence of the n -point vertex depends on the $n+1$ and the $n+2$ point vertex. In practical computations, the tower of scale-dependent couplings is therefore truncated by identifying the couplings appearing in the $n+1$ and $n+2$ -point vertices with those of the n -point vertex.

该展开的第一项仅依赖于背景度规，我们将这一项中的耦合称为背景耦合。这些耦合是最终出现在可观测量中的物理耦合。式 (3) 中的第二项是对 n 点顶点的求和，这些顶点的标度依赖通常与背景耦合不同。我们将这些 n 点顶点中的耦合称为涨落耦合。从式 (3) 和流方程式 (1) 可以看出，n 点顶点的标度依赖依赖于 $n+1$ 和 $n+2$ 点顶点。因此在实际计算中，人们通过将 $n+1$ 和 $n+2$ 点顶点中的耦合等同于 n 点顶点的耦合，对依赖标度的耦合塔进行截断。

As an alternative to the expansion in background metric and fluctuation field, one can also work in a bimetric setting, with background metric and full metric. For gravity-matter systems, this has been implemented in [37], but a majority of works in this context uses the fluctuation field instead. A main reason for this choice is that the scale dependence of the fluctuation couplings can be extracted by choosing a flat background metric, which is technically advantageous.

除了按背景度规和涨落场展开，也可以在双度规框架下工作，同时引入背景度规和完整度规。对于引力-物质系统，该方案已在文献 [37] 中实现，但该领域的大多数工作仍使用涨落场展开。做出这一选择的主要原因是，选取平直背景度规即可提取涨落耦合的标度依赖，在技术上更具优势。

While the physical couplings are only contained in $\Gamma_k[\bar{g}, 0]$, their scale dependence is driven by the fluctuation couplings. The background-field approximation only computes the scale dependence of these physical couplings and identifies all fluctuation couplings with the corresponding background coupling. In particular, the momentum-independent part of the two-point function is identified with the cosmological constant $\overline{\Lambda}$. This approximation scheme allows computing the scale-dependence of curvature operators of high powers (see, e.g., [38-40]) and even of form factors (see, e.g., [41,42]). It also allows to extract the scale dependence on nontrivial and even arbitrary backgrounds; see, e.g., [43-45]. The background-field approximation has the virtue of relying on background diffeomorphism invariance by extracting the scale dependence at vanishing fluctuation field h . However, it neglects the difference between the fact that the full effective action individually depends on the background metric $\bar{g}$ and metric fluctuations h and therefore assumes a trivial realization of the so-called Nielsen identities.

尽管物理耦合仅包含在 $\Gamma_k[\bar{g}, 0]$ 中, 它们的标度依赖由涨落耦合驱动。背景场近似只计算这些物理耦合的标度依赖，并将所有涨落耦合等同于对应的背景耦合。具体而言，两点函数中不依赖动量的部分被等同于宇宙学常数 $\overline{\Lambda}$ 。该近似方案可以计算高次曲率算符的标度依赖 (参见例如文献 [38-40]), 甚至可以计算形状因子的标度依赖 (参见例如文献 [41,42])。它也可以提取非平凡甚至任意背景上的标度依赖；参见例如文献 [43-45]。背景场近似的优点在于，通过在涨落场 h 为零处提取标度依赖，它依赖于背景微分同胚不变性。但该近似忽略了全有效作用量分别依赖于背景度规 $\bar{g}$ 和度规涨落 h 这一差异，因此假定所谓的尼尔森恒等式是平凡实现的。

Fluctuation computations focus on the computation of the scale dependences included in the second term of Eq. (3). The tower of n -point vertices is typically truncated at some order m , and the couplings appearing in the $m+1$ and $m+2$ - point vertices are identified with those of the m -point vertex.

涨落计算的核心是计算式 (3) 第二项中包含的标度依赖关系。n 点顶点的层级通常会在 m 阶截断，$m+1$ 点顶点与 $m+2$ 点顶点中出现的耦合被认为等同于 m 点顶点的耦合。

Fundamentals

基础概念

There are a few notions that will be key to this whole chapter. We discuss these fundamentals here.

本章全程会用到几个核心概念，我们在此讨论这些基础内容。

The Direction of the RG Flow

RG 流的方向

Which direction of the RG flow should we think of, when talking about asymptotic safety? It is tempting to think about the RG flow from IR to UV, because in this way we extrapolate from known and measured physics into the unknown. Indeed, asymptotic safety is sometimes discussed in this way. However, this direction is not physically meaningful, because in nature, microphysics determines macrophysics and not the other way around. Thus, a meaningful RG flow always starts at high energies and goes toward low energies, where the high-energy physics has low-energy consequences.

在谈论渐近安全时，我们应当考虑哪个方向的 RG 流？人们很容易会思考从红外到紫外的 RG 流，因为通过这种方式我们可以从已知的、已测量的物理外推到未知领域。实际上，渐近安全有时确实是以这种方式讨论的。但这个方向并不具有物理意义，因为在自然界中，微观物理决定宏观物理，而非反过来。因此，一个有物理意义的 RG 流总是从高能出发，流向低能，高能物理会在低能产生效应。

Quantum Scale Symmetry: What Is Constant?

量子标度对称性: 什么是恒定不变的?

Asymptotic safety is an enhancement of the symmetry of the QFT. The added symmetry is quantum scale symmetry, which means that couplings are nonvanishing and constant when the energy scale is changed. In a classical field theory, requiring constant couplings would be a trivial requirement; in a quantum theory, it is not. In a quantum theory, quantum fluctuations screen or antiscreen couplings and thereby generate a scale dependence. Asymptotic safety means that this scale dependence vanishes in the dimensionless counterparts of couplings. This is a key difference to what is often referred to as scale invariance in the literature. Because scale invariance is, loosely translated, the absence of distinct physical scales, scale invariance is often taken to mean that there cannot be dimensionful couplings in the theory. This statement is true in quantum scale symmetry, but in a more subtle way: when a dimensionful coupling vanishes in the UV, $\bar{g} = 0$, then its dimensionless counterpart, $g = \bar{g}k^{-d_{\bar{g}}}$, need not vanish: the RG scale k is taken to make g dimensionless;

and when, e.g., the dimension of the coupling, $d_{\bar{g}}$, is negative, then $\bar{g}$ vanishes in the limit $k \to \infty$, even if g is nonzero in that limit. This is the sense in which quantum scale symmetry is a scale symmetry: the dimensionful quantities scale to zero (or infinity) if one takes the formal limit $k \to \infty$, so that no scales are present, cf. Fig. 3. Nevertheless, the dimensionless counterparts of these dimensionful quantities remain nonvanishing and constant. In this sense, asymptotic safety is a generalization of asymptotic freedom, which is familiar from quantum chromodynamics. In asymptotic freedom, the coupling vanishes in the $k \to \infty$ limit. In those systems, classical scale invariance is restored, since the classical theory only contains dimensionless quantities and quantum fluctuations vanish for $k \to \infty$.

渐近安全是量子场论对称性的扩充。新增的对称性就是量子标度对称性，它意味着当能标改变时，耦合保持非零且恒定。在经典场论中，要求耦合恒定是平凡的要求；但在量子理论中并非如此。量子理论里，量子涨落会对耦合产生屏蔽或反屏蔽作用，进而产生标度依赖性。渐近安全指的是，这种标度依赖性在耦合的无量纲对应量中消失。这和文献中通常所说的标度不变性有一个关键区别。粗略来说，标度不变性意味着不存在独立的物理标度，因此通常认为标度不变性要求理论中不能存在量纲耦合。这个结论在量子标度对称性中也成立，但形式更微妙: 当一个量纲耦合在紫外区消失时，$\bar{g} = 0$ ，那么它的无量纲对应量 $g = \bar{g}k^{-d_{\bar{g}}}$ 不一定为零: 重整化群标度 k 被用来将 g 转化为无量纲形式；例如，当耦合的量纲 $d_{\bar{g}}$ 为负时，即使 g 在极限 $k \to \infty$ 下非零，$\bar{g}$ 也会在该极限下消失。从这个意义上来说，量子标度对称性就是一种标度对称性: 当取形式极限 $k \to \infty$ 时，量纲量会标度到零 (或无穷大)，因此理论中不存在任何标度，参见图 3。尽管如此，这些量纲量的无量纲对应量仍然保持非零且恒定。从这个意义上说，渐近安全是量子色动力学中人们熟知的渐近自由的推广。在渐近自由中，耦合在 $k \to \infty$ 极限下消失。在这类系统中，经典标度不变性得以恢复，因为经典理论仅包含无量纲量，且量子涨落在 $k \to \infty$ 下消失。

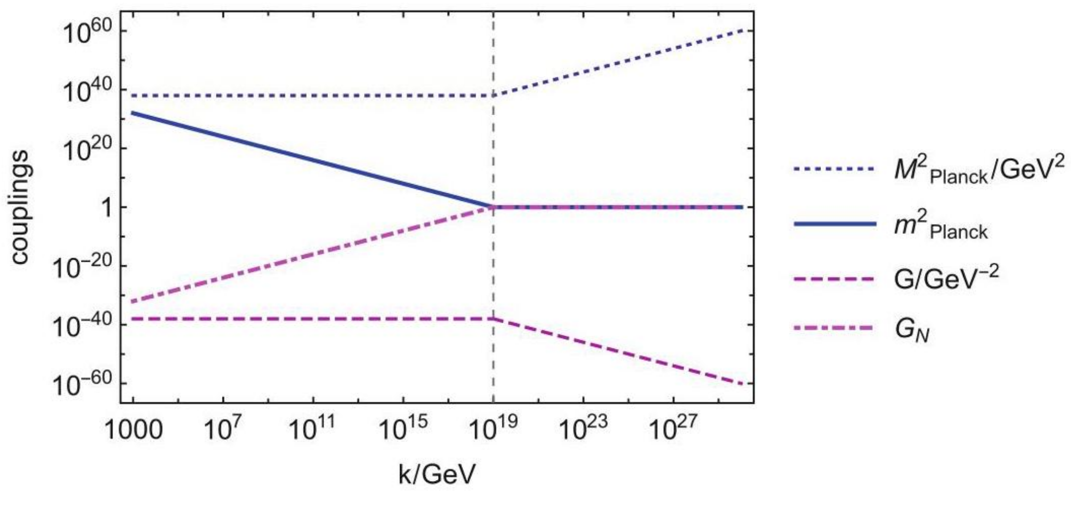


Fig. 3 We show the dimensionful Planck mass (M^2_{Planck} and its dimensionless counterpart $m^2_{\mathrm{Planck}} = M^2_{\mathrm{Planck}}/k^2$), as well as the dimensionful Newton coupling G and its dimensionless counterpart $G_N = G \cdot k^2$. Beyond the Planck scale, at 10^{19}GeV , the dimensionless quantities are constant and the dimensionful ones show scaling behavior. In the limit $k \to \infty$, the theory therefore emerges from a regime without scales, because all masses scale to infinity. Thinking about the theory from the IR to the UV, as is sometimes done, one expects a strong coupling regime to set in at the Planck scale. Instead, a scaling regime sets in, which is characterized by G_N decreasing. Thus, in asymptotic safety, gravity is more weakly coupled than expected, which can be viewed as a reason why the continuum spacetime picture for the gravitational interaction continues to hold and does not need to be substituted by a description of radically different character. Similarly,

the Planck mass, at which one expects gravity to become strongly coupled, ”runs away,” once $k^2 = 10^{19}$GeV is reached, i.e., the theory may dynamically protect itself from strong coupling phenomena

图 3 我们展示了有量纲普朗克质量 (M^2_{Planck} 及其无量纲对应量 $m^2_{\text{Planck}} = M^2_{\text{Planck}}/k^2$ ，还有量纲牛顿耦合 G 及其无量纲对应量 $G_N = G \cdot k^2$ 。在普朗克标度之外，也就是 10^{19}GeV 处，无量纲量保持恒定，量纲量呈现标度行为。因此在极限 $k \to \infty$ 下，理论从一个无标度区域演生出来，因为所有质量都会标度到无穷大。如果像有时那样从红外往紫外思考理论，人们会认为强耦合区会在普朗克标度出现。但实际上出现的是标度区，其特征是 G_N 不断减小。因此在渐近安全框架下，引力比预期的更弱耦合，这也可以解释为什么引力相互作用的连续时空图像仍然成立，不需要被性质完全不同的描述替代。类似地，人们原本预期引力会变成强耦合的普朗克质量会“逃逸”，一旦达到 $k^2 = 10^{19}$GeV ，也就是说，理论可以通过动力学方式保护自己，不出现强耦合现象

Predictions from asymptotic safety

渐近安全的预言

Because asymptotic safety is an enhancement of the symmetry of the QFT, one may expect that it relates different interactions to each other, because this is what happens in a QFT, when an additional symmetry is imposed. However, the special property of quantum scale symmetry is that such relations between couplings can remain intact, even if the theory departs from quantum scale symmetry under the RG flow toward the infrared (IR). In fact, the simple observation that there are distinct scales in nature, e.g., those of the masses of elementary particles, means that the RG flow must leave the asymptotically safe fixed point regime at some scale, i.e., quantum scale symmetry can only hold in the UV, not the IR; see, though, [46] for an alternative. Nevertheless, the IR physics can still carry imprints of quantum scale symmetry in relations between couplings. We now exemplify this property. In short, it stems from the fact that the RG flow has ”sources” and ”sinks” - in technical language, relevant and irrelevant directions. It can depart from the asymptotically safe fixed-point regime along a ”source direction” but not a ”sink direction.” In ”sink directions,” quantum fluctuations generate scale symmetry on the way to the IR, i.e., even if one chooses the value of a coupling slightly away from the fixed point, quantum fluctuations drive the coupling back to the fixed point. Therefore, along a ”sink direction,” there is only one value that the coupling can take in the IR, namely, its fixed-point value. This is true even if the RG flow has departed from the fixed-point value along a ”source direction”; see left panel in Fig. 4.

由于渐近安全是量子场论的对称性增强，可以预期它会将不同的相互作用关联起来——因为当量子场论被附加额外对称性时，就会发生这种情况。而量子标度对称性的特殊性质在于，即使理论在流向红外 (IR) 的重整化群演化中偏离了量子标度对称性，耦合之间的这类关系仍然可以保持不变。实际上，自然界存在不同尺度 (例如基本粒子的质量尺度) 这一简单观测就表明，重整化群流必定会在某个尺度离开渐近安全固定点区域，也就是说量子标度对称性只能在紫外 (UV) 成立，而非红外；不过关于另一种观点参见文献 [46]。尽管如此，红外物理依然会在耦合关系中留下量子标度对称性的印记。我们接下来举例说明这一性质。简言之，它源于重整化群流存在「源」和「汇」——用专业术语来讲就是相关方向和无关方向。重整化群流只能沿「源方向」离开渐近安全固定点区域，无法沿「汇方向」离开。在「汇方向」上，量子涨落会在流向红外的过程中重建标度对称性: 也就是说，即使人为将一个耦合的取值选在稍微偏离固定点的位置，量子涨落也会将该耦合推回固定点。因此，沿「汇方向」，耦合在红外只能取一个值，也就是它的固定点值。哪怕重整化群流已经沿「源方向」偏离了固定点值，这一点依然成立；参见图 4 左图。

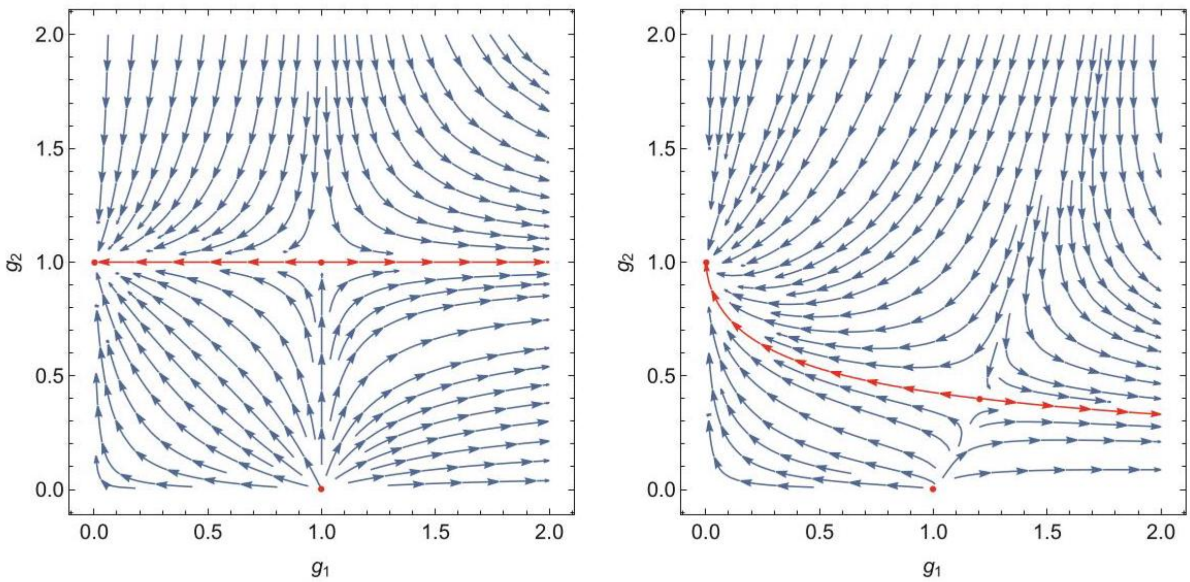


Fig. 4 The schematic RG flow has three fixed points in both panels. The one at ($g_1 = 1, g_2 = 0$) acts as a source in both directions and thus does not generate predictions. The one at ($g_1 = 0, g_2 = 1$) acts as a sink in both directions. It therefore generates two predictions but also requires scale symmetry at all scales, because the RG flow cannot depart from it. Finally, the point at ($g_1 = 1$, $g_2 = 1$) (left panel) and $g_1 = 1.2, g_2 = 0.4$ (right panel) has one "source direction" and one "sink direction." In both cases, it generates a prediction of the value of g_2 . In the simple case (left panel), the prediction is the fixed-point value. In the more realistic case (right panel), the prediction is a function of g_1

图 4 示意性的重整化群流在两个分图中都各有三个固定点。位于 ($g_1 = 1, g_2 = 0$) 的固定点在两个方向上都是源，因此不会产生预言。位于 ($g_1 = 0, g_2 = 1$) 的固定点在两个方向上都是汇，因此它会产生两个预言，但也要求所有尺度都满足标度对称性，因为重整化群流无法离开它。最后，位于 ($g_1 = 1$, $g_2 = 1$)(左图) 和 $g_1 = 1.2, g_2 = 0.4$ (右图) 的点拥有一个「源方向」和一个「汇方向」。两种情况下，它都会对 g_2 的取值给出预言。在简单情形 (左图) 中，预言就是固定点值。在更现实的情形 (右图) 中，预言是 g_1 的函数。

In practice, the simple picture we just sketched out is slightly modified in terms of the quantitative aspects of the predictions: first, when a relevant coupling departs from its fixed-point value, it can pull an irrelevant coupling along with it. The special value that the irrelevant coupling is fixed to is then no longer the fixed-point value but instead a value that depends on the relevant coupling. In technical language, this means that the critical hypersurface of the fixed point (spanned by all its ”source directions” is curved). Second, at an asymptotically safe fixed point, it is often a superposition of couplings that corresponds to a ”source” or a ”sink direction.”

实际应用中，我们刚刚勾勒出的简单图像在预言的定量层面会有一点修正: 首先，当一个相关耦合偏离它的固定点值时，会带动一个无关耦合一同偏离。此时无关耦合被固定到的特殊值就不再是固定点值，而是依赖于该相关耦合的一个值。用专业术语来讲，这意味着固定点的临界超曲面 (由所有「源方向」张成) 是弯曲的。其次，在渐近安全固定点处，「源方向」或「汇方向」通常对应多个耦合的叠加。

Both aspects combined means that one often gets relations between couplings that are predicted at low energies. In practice, these can often only be calculated numerically.

两个特点结合起来意味着，我们通常会得到低能下被预言的耦合之间的关系，这些关系实际上往往只能通过数值计算得到。

In more technical language, predictions arise from irrelevant directions of the RG flow, which are encoded in negative critical exponents. The critical exponents measure the changes of the flow, i.e., they are related to first derivatives of the beta function. Thereby, they encode whether a direction corresponds to a source or sink. The critical exponents are calculated as

用更专业的语言来说，预言来源于重整化群流的无关方向，这些方向由负临界指数编码。临界指数衡量流的变化，也就是说它们与贝塔函数的一阶导数相关，借此可以编码一个方向对应源还是汇。临界指数的计算公式为

$$\theta_I = -\,\mathrm{eig}\left(\frac{\partial \beta_{g_i}}{\partial g_j}\right)\Bigg|_{g_i=g_{i*}}, \tag{4}$$

where g_{i*} is the fixed-point value of a coupling. Because of the extra negative sign, $\theta_I > 0$ corresponds to a source (a relevant direction) and $\theta_I < 0$ to a sink (an irrelevant direction).

其中 g_{i*} 是耦合的固定点值。由于额外的负号，$\theta_I > 0$ 对应源 (相关方向)，$\theta_I < 0$ 对应汇 (无关方向)。

Partial vs. Full Fixed Points

部分固定点 vs 全固定点

A fixed point is a point where the scale dependence of all couplings g_i vanishes, i.e., $\beta_{g_i} = 0$ for all couplings g_i of the system. However, deciding whether a fixed point exists is a major challenge, because it requires knowledge of all beta functions. In practice, we will nevertheless refer to, e.g., the gravitational fixed point, because a large number of couplings have been included in studies and there are no indications that the remaining couplings would spoil the fixed point in gravity.

固定点是指系统中所有耦合 g_i 的标度依赖都消失的点，即对系统的所有耦合 g_i 都满足 $\beta_{g_i} = 0$。然而，判断一个固定点是否存在是一项重大挑战，因为这需要掌握所有贝塔函数的信息。但在实践中，我们仍然会讨论例如引力固定点这类概念，因为已有研究纳入了大量耦合，且没有迹象表明剩余耦合会破坏引力中的固定点。

In the following, and in particular in sections ”Global Symmetries Persist and Have Phenomenological Consequences” and ”Toward a UV Completion of the Standard Model”, we will at times investigate the fixed-point structure of a subsystem while treating other couplings as external parameters. This allows us to factorize the search for an asymptotically safe fixed-point for gravity and matter into different subsystems. Strictly speaking, we then search for partial fixed points within these different subsystems. The fixed points are partial, because some couplings are treated as external parameters, instead of also being evaluated at their respective fixed point.

在下文中，尤其是在“全局对称性持续存在并具有现象学意义”和“迈向标准模型的紫外完备化”这两节中，我们会不时研究子系统的固定点结构，同时将其他耦合作为外参数处理。这让我们可以把寻找引力与物质的渐近安全固定点分解到不同子系统中完成。严格来说，我们此时是在这些不同子系统内搜索部分固定点。这些固定点被称为部分固定点，是因为部分耦合被当作外参数处理，而没有也放到它们各自的固定点上进行计算。

The existence of a partial fixed point is a necessary, but not sufficient condition to find a fixed point of the full system. We will refer to partial fixed points as fixed points for coupling g_i , while full fixed points will be referred to as fixed points of the full system.

部分固定点的存在是找到全系统固定点的必要非充分条件。我们将耦合 g_i 的部分固定点简称为耦合 g_i 的固定点，而全系统的固定点则称为全固定点。

The Robustness of Results

结果的稳健性

How sure are we about results in asymptotically safe gravity-matter systems? The answer differs, depending on which result we have in mind. There is no result about the existence of an asymptotically safe fixed point in gravity-matter systems which has been proven in a strict mathematical sense. However, for some results, so much evidence has been accumulated that they can be assumed to hold beyond reasonable doubt. For others, the uncertainty is larger, because the effect of approximations and assumptions is less well understood. We try to highlight when this is the case, without making our text too cluttered by constantly repeating phrases like ”within an approximation,” ”under certain assumptions,” etc.

我们对渐近安全引力-物质系统的研究结果有多大把握？答案取决于我们所讨论的具体结果。目前引力-物质系统中存在渐近安全不动点这一结论还没有得到严格数学意义上的证明。但对于部分结果而言，现已积累的充足证据足以让我们认定它们成立，不存在合理怀疑。另一些结果则存在更大的不确定性，因为我们对近似和假设带来的影响还不够了解。我们会对这类情况加以说明，不会在文本中不断重复“在某一近似下”“在特定假设下”这类表述，导致内容过于冗杂。

There is a way of testing the robustness of results that we will refer to: if an approximation is advanced enough, then unphysical choices (for instance, choices of gauge parameters) do not change the value of physical quantities at all or not much. Conversely, if an approximation is not advanced enough, the choice of unphysical parameters can start to matter.

我们采用一种检验结果稳健性的方法: 如果近似足够完善，那么非物理选择 (例如规范参数的选择) 完全不会或仅会很小程度改变物理量的取值；相反，如果近似不够完善，非物理参数的选择就会对结果产生影响。

Gravitational Fixed Point Under the Impact of (Minimally Coupled) Matter

(最小耦合) 物质影响下的引力不动点

There is compelling evidence for an asymptotically safe fixed point in pure gravity; see, e.g., other sections in this handbook. This fixed point is the starting point for our discussion. We will explore whether and how this fixed point continues to exist, when more and more matter fields of different spins are added. In this section, the self-interactions of matter are ignored, because they do only indirectly impact the gravitational fixed point, although they may or may not have a fixed point themselves.

纯引力中存在渐近安全不动点这一点已有令人信服的证据，例如参见本手册其他章节。该不动点是我们讨论的出发点。我们将探究当加入越来越多不同自旋的物质场时，这个不动点是否还能继续存在，以及它会如何变化。在本节中，我们忽略物质的自相互作用，因为自相互作用仅对引力不动点产生间接影响，尽管物质自相互作用本身可能存在也可能不存在不动点。

Screening and Antiscreening Effects of Matter on the Newton Coupling

物质对牛顿耦合的屏蔽与反屏蔽效应

Synopsis: Matter fields of different spins have different effects on the fixed point in the Newton coupling: scalars and fermions disfavor it; gauge fields favor it. This can be understood in terms of screening and antiscreening contributions, i.e., weakening and strengthening of gravity.

提要: 不同自旋的物质场对牛顿耦合定点的作用各不相同: 标量场与费米子不利于定点存在；规范场则支持定点存在。这可以从屏蔽和反屏蔽贡献，也就是引力的削弱与增强来理解。

Asymptotic safety arises when there is an overall antiscreening contribution of quantum fluctuations of all fields - matter and gravitational. An antiscreening contribution is one with a negative sign in the beta function. Such a contribution is necessary to achieve quantum scale symmetry, because the canonical dimension of the Newton coupling generates a contribution with positive sign. An antiscreening contribution can compensate this, such that asymptotic safety is present. Note that there is a way of understanding why an antiscreening contribution is necessary which uses continuation of the theory across dimensions: in $d = 2$, the Newton coupling is dimensionless, which is similar to the gauge coupling in $d = 4$: just like screening effects mean that the Abelian gauge coupling is screened to zero in four-dimensional QED, screening effects would mean a nongravitating gravity theory. In contrast, just like antiscreening effects mean that the non-Abelian gauge coupling is antiscreened to a nonzero value in four-dimensional QCD, antiscreening effects yield a gravity theory with nonvanishing gravitational interaction. In the UV, this theory is asymptotically free, i.e., the coupling starts out at zero in the UV and is antiscreened to nonzero in the IR. Going from $d = 2$ to $d > 2$, the antiscreening contribution can remain but must compete with a contribution from a positive sign from the canonical dimension of the coupling, such that asymptotic freedom is no longer available, but asymptotic safety is.

当包括物质场和引力场在内的所有场的量子涨落整体产生反屏蔽贡献时，渐近安全就会出现。反屏蔽贡献是贝塔函数中符号为负的贡献。这类贡献是实现量子标度对称性的必要条件，因为牛顿耦合的正则维度会产生一个符号为正的贡献。反屏蔽贡献可以抵消这一正贡献，从而实现渐近安全。值得注意的是，可以通过跨维延拓来理解为什么反屏蔽贡献是必要的: 在 $d = 2$ 中，牛顿耦合是无量纲量，这与 $d = 4$ 中规范耦合的情况类似: 正如屏蔽效应意味着四维 QED 中阿贝尔规范耦合被屏蔽到零，屏蔽效应会得到一个无引力的引力理论。反之，正如反屏蔽效应意味着四维 QCD 中非阿贝尔规范耦合被反屏蔽到非零值，反屏蔽效应会得到一个引力相互作用不为零的引力理论。该理论在紫外是渐近自由的，即耦合在紫外初始为零，在红外被反屏蔽到非零值。从 $d = 2$ 到 $d > 2$ ，反屏蔽贡献可以保留，但必须与耦合正则维度带来的正贡献竞争，因此渐近自由不再成立，渐近安全得以成立。

When gravity is coupled to N_{S} scalar fields, N_{F} Dirac fermions, and N_{V} vector fields, the scale dependence of the Newton coupling can be schematically written as

当引力与 N_{S} 个标量场、N_{F} 个狄拉克费米子和 N_{V} 个矢量场耦合时，牛顿耦合的标度依赖可以示意性地写为

$$\beta_{G_{\mathrm{N}}} = 2G_{\mathrm{N}} - G_{\mathrm{N}}^2 \left(b_{\text{grav}} + a_{\mathrm{S}}N_{\mathrm{S}} + a_{\mathrm{F}}N_{\mathrm{F}} + a_{\mathrm{V}}N_{\mathrm{V}}\right) + \mathcal{O}\left(G_{\mathrm{N}}^3\right), \tag{5}$$

where the first term encodes the scale dependence due to the canonical mass dimension of the Newton coupling which leads to perturbative nonrenormalizability. Gravitational fluctuations and Faddeev-Popov ghosts generate $b_{\text{grav}} > 0$; see, e.g., $[36, 38, 40, 44, 47 - 76]$, as well as [77] and references therein. The coefficients a_{S} ($a_{\mathrm{F}}, a_{\mathrm{V}}$) encode whether scalar (fermionic, vector) fields screen ($a_i < 0$) or antiscreen ($a_i > 0$) the gravitational coupling. Note that more precisely matter fields also impact all other gravitational couplings, e.g., the cosmological constant; see section "The Effective Strength of Gravity Under the Impact of Matter". For the case of minimally coupled matter fields, the a_i are numerical factors [78-94]; going beyond minimal coupling, the a_i become functions of the couplings (see, e.g., [91, 95-100]) and have to be evaluated at the corresponding fixed-point values.

其中第一项编码了由牛顿耦合的正则质量维度带来的标度依赖，该正则质量维度导致了微扰不可重整性。引力涨落和法捷耶夫-波波夫鬼场产生了 $b_{\text{grav}} > 0$；参见例如 [36, 38, 40, 44, 47 – 76]，以及文献 [77] 及其中的参考文献。系数 a_S (a_F, a_V) 体现了标量 (费米、矢量) 场对引力耦合是产生屏蔽 ($a_i < 0$) 还是反屏蔽 ($a_i > 0$)。注意，更准确地说物质场也会影响所有其他引力耦合，例如宇宙学常数；参见章节“物质影响下引力的有效强度”。对于最小耦合物质场，a_i 是数值因子 [78-94]；超出最小耦合后，a_i 变为耦合的函数 (参见例如 [91, 95-100])，必须在对应定点值处计算。

The resulting gravitational fixed-point value is

最终得到的引力定点值为

$$G_{N,*} = \frac{2}{b_{\text{grav}} + a_S N_S + a_F N_F + a_V N_V}. \tag{6}$$

Equation 6 shows that a screening contribution ($a_i < 0$) increases the fixed-point value of the Newton coupling $G_{N,*}$, while an antiscreening contribution ($a_i > 0$) decreases it. If the screening contributions dominate over the antiscreening contributions, then $G_{N,*} \to \infty$ (and subsequently $G_{N,*} < 0$) and the theory is not asymptotically safe. Thus, screening contributions destabilize the asymptotically safe gravity system, because they can overcome the gravitational contribution b_{grav} and thereby remove the interacting fixed point. Conversely, an antiscreening contribution $a_i > 0$ stabilizes the system and drives the system to $G_{N,*} \to 0$, when increasing the corresponding N_i. Note that this does not necessarily indicate that a perturbative fixed point is approached, because fixed-point values of couplings can always be made arbitrarily small by an appropriate rescaling. Instead, the critical exponents are a meaningful measure of perturbativity: if they approach the canonical values, the fixed point becomes perturbative. In the present case, the critical exponent of the Newton coupling in the approximation (5) is always 2, independent of the fixed-point value.

式 (6) 表明，屏蔽贡献 ($a_i < 0$) 会增大牛顿耦合 $G_{N,*}$ 的不动点值，而反屏蔽贡献 ($a_i > 0$) 会减小该值。若屏蔽贡献占优于反屏蔽贡献，则会得到 $G_{N,*} \to \infty$ (进而得到 $G_{N,*} < 0$)，理论不满足渐近安全。因此，屏蔽贡献会破坏渐近安全引力系统的稳定性，因为它会超过引力贡献 b_{grav}，从而消除相互作用不动点。相反，当增大对应 N_i 时，反屏蔽贡献 $a_i > 0$ 会稳定系统并推动系统向 $G_{N,*} \to 0$ 演化。请注意，这并不一定意味着系统会趋近微扰不动点，因为通过合适的标度变换总能让耦合的不动点值变得任意小。而临界指数才是微扰性的有效度量: 若临界趋近经典值，不动点就成为微扰不动点。在本例中，近似 (5) 下牛顿耦合的临界指数始终为 2，与不动点值无关。

In principle, there are different ways of coupling matter to gravity - minimally and nonminimally. For minimally coupled fields, the interaction with gravity lies in the kinetic term of the matter fields. For nonminimally coupled fields, explicit couplings with curvature terms are present. In classical or phenomenological studies, one can usually choose which coupling to include. In asymptotically safe gravity-matter systems, there is no such choice to make: all interactions compatible with the symmetries are generically present, and this includes some nonminimal interactions. Nevertheless, their effect does not need to be significant, because the nonminimal couplings may be small at a fixed point. Indeed, in studies to date, the minimal coupling determines whether matter fields screen or antiscreen the gravitational fixed point.

原则上，物质与引力有不同的耦合方式: 最小耦合与非最小耦合。最小耦合场中，物质与引力的相互作用出现在物质场的动能项中；非最小耦合场中则存在与曲率项的显式耦合。在经典或唯象研究中，通常可以选择包含哪些耦合；在渐近安全引力-物质系统中，不存在这种选择空间: 所有与对称性相容的相互作用一般都会存在，其中包括部分非最小相互作用。但这类相互作用的影响未必显著，因为非最小耦合在不动点处可能很小。迄今为止的研究确实表明，最小耦合决定了物质场对引力不动点是产生屏蔽还是反屏蔽作用。

Minimally coupled scalars screen the gravitational coupling, $a_S < 0$; see [45, 78, 80 − 84, 86, 88, 89, 99, 101, 102] . These studies cover a range of different approximations and technical choices, e.g., regulator function and gauge parameters. The sign of a_S can thus be regarded as settled. A screening contribution was also found using other methods, namely, perturbative heat kernel methods [103,104] and an ε expansion around $2 + \varepsilon$ dimensions [7].

最小耦合标量场会屏蔽引力耦合，$a_S < 0$；参见 [45, 78, 80 − 84, 86, 88, 89, 99, 101, 102] 。这些研究涵盖了一系列不同的近似和技术选择，例如调节函数和规范参数。因此 a_S 的符号已有定论。使用其他方法也发现了屏蔽贡献，即微扰热核方法 [103,104] 和围绕 $2 + \varepsilon$ 维的 ε 展开 [7]。

Minimally coupled fermions also screen the gravitational coupling, $a_F < 0$. This was found in various studies which cover different approximations, regulator functions, and gauge parameters [45, 79, 80, 82, 85, 88, 90, 91, 93, 94, 102 and also in perturbative studies [103, 104].

最小耦合费米子也会屏蔽引力耦合，$a_F < 0$ 。这一结论在覆盖不同近似、调节函数和规范参数 [45, 79, 80, 82, 85, 88, 90, 91, 93, 94, 102] 的多项研究中都得到了验证，在微扰研究 [103,104] 中也成立。

Minimally coupled gauge fields antiscreen the gravitational coupling, $a_V > 0$, as was found in various studies [45, 80, 86 − 88, 102] , in agreement with perturbative studies [103, 104]. Because gauge fields antiscreen the Newton coupling, one may expect that the gravitational fixed point becomes the free fixed point for $N_V \to \infty$. Indeed, the fixed-point value $G_{N,*}$ approaches zero for increasing N_V . However, the fixed-point value for the cosmological constant remains nonzero [80,87]; therefore, the limit $N_V \to \infty$ does not lead to an asymptotically free fixed point.

多项研究 [45, 80, 86 − 88, 102] 均发现，最小耦合规范场会反屏蔽引力耦合，$a_V > 0$，这与微扰研究 [103,104] 的结论一致。由于规范场反屏蔽牛顿耦合，可以预期当 $N_V \to \infty$ 时，引力不动点会转变为自由不动点。事实上，随着 N_V 增大，不动点值 $G_{N,*}$ 会趋近于零。但宇宙常数的不动点值仍保持非零 [80,87]；因此，$N_V \to \infty$ 的极限并不会产生渐近自由不动点。

Further Reading

延伸阅读

On the Impact of Nonminimal Couplings

论非最小耦合的影响

Some studies have gone beyond minimal coupling and included explicit couplings between matter fields and curvature terms; see Table 1 for an overview and references.

已有部分研究超出了最小耦合的范畴，将物质场与曲率项之间的显式耦合纳入研究；综述与参考文献参见表 1。

A nonminimal coupling between scalars and gravity of the form $R^{\mu\nu}D_\mu\phi D_\nu\phi$ slightly increases the amount of screening [99], while a nonminimal coupling of the form $RD_\mu\phi D^\mu\phi$ slightly reduces the amount of screening [99]. The nonminimal coupling $\phi^2 R$, which is canonically marginal, breaks shift symmetry and therefore, as we shall explain in section ”Global Symmetries Persist and Have Phenomenological Consequences”, is allowed to vanish [78,81,83,95,105], unless shift symmetry is broken through the presence of, e.g., Yukawa interactions; see [106].

$R^{\mu\nu}D_\mu\phi D_\nu\phi$ 形式的标量与引力非最小耦合会小幅提升屏蔽量 [99]，而 $RD_\mu\phi D^\mu\phi$ 形式的非最小耦合会小幅降低屏蔽量 [99]。正则边际的非最小耦合 $\phi^2 R$ 会破缺平移对称性，因此正如我们会在“整体对称性存续且具有唯象学后果”一节中解释的，除非平移对称性因汤川相互作用等因素已经破缺，否则该耦合可以取零值 [78,81,83,95,105]；参见文献 [106]。

A nonminimal coupling between fermions and gravity of the form $R\psi\overline{\psi}$ slightly increases the amount of screening [96]. On the other hand, a nonminimal coupling of the form $R^{\mu\nu}\overline{\psi}\gamma_\mu\nabla_\nu\psi$ reduces the amount of screening and therefore stabilizes the system [91]. (We motivate the importance of such a nonminimal coupling in section ”Global Symmetries Persist and Have Phenomenological Consequences”.) However, even in the nonminimally coupled fermion-gravity systems, fermions screen the Newton coupling.

$R\psi\overline{\psi}$ 形式的费米子与引力非最小耦合会小幅提升屏蔽量 [96]。另一方面，$R^{\mu\nu}\overline{\psi}\gamma_\mu\nabla_\nu\psi$ 形式的非最小耦合会降低屏蔽量，从而稳定系统 [91]。(我们会在“整体对称性存续且具有唯象学后果”一节中说明这类非最小耦合的重要性。) 然而，即便是在非最小耦合的费米子-引力系统中，费米子仍然会对牛顿耦合产生屏蔽。

For vectors, nonminimal interactions have not yet been investigated.

针对矢量场的非最小相互作用尚未有研究。

Table 1 We list the interactions that were included in the studies in the corresponding references. We only list the most comprehensive studies at each set of interactions, i.e., for the Einstein-Hilbert truncation, on which the impact of all three species of matter fields was studied, we do not separately list studies which only take into account one or two species of matter fields. Those nonminimal couplings marked by an * cannot be set to zero at an interacting gravitational fixed point; see section ”Global Symmetries Persist and Have Phenomenological Consequences”. In contrast, those nonminimal couplings not marked by an * can be set to zero for reasons of symmetry; thus, it is consistent to neglect them at a minimally coupled fixed point. We also indicate the numbers of matter fields that were studied, where ”arb.” stands for an arbitrary number of fields of the corresponding species but does not necessary imply that an asymptotically safe fixed point exists for arbitrarily high numbers of the corresponding field. We omit matter self-interactions in this table; those are discussed in section ”Global Symmetries Persist and Have Phenomenological Consequences”

表 1 本表列出了对应参考文献的研究中纳入的相互作用。我们仅列出每种相互作用类别下最全面的研究，例如对于研究了三类物质场影响的爱因斯坦-希尔伯特截断，我们不再单独列出仅考虑一类或两类物质场的研究。带 * 标记的非最小耦合无法在相互作用引力不动点处取零值；参见“整体对称性存续且具有唯象学后果”一节。反之，不带 * 标记的非最小耦合因对称性可以取零值，因此在最小耦合不动点处忽略它们是自洽的。我们同时标注了研究中使用的物质场数目，其中“arb.”代表对应种类场的数目任意，不代表渐近安全不动点一定存在于任意多数目的对应场中。本表省略了物质自相互作用，这类相互作用将在“整体对称性存续且具有唯象学后果”一节中讨论

Gravitational int.’s	Nonminimal int.’s of scalars	Nonminimal int.’s of fermions	Nonminimal int.’s of gauge fields	N_S	N_F	N_V
$\sqrt{g}, \sqrt{g}R$ [80, 86, 102]	-	-	-	arb.	arb.	arb.
$\sqrt{g}, \sqrt{g}f(R)$ [88]	-	-	-	arb.	arb.	arb.
$\sqrt{g}, \sqrt{g}R, \sqrt{g}R^2\ \sqrt{g}C_{\mu\nu\kappa\lambda}C^{\mu\nu\kappa\lambda}$ [45]	-	-	-	arb.	arb.	arb.
$\sqrt{g}, \sqrt{g}R\ \sqrt{g}R^2, \sqrt{g}R_{\mu\nu}R^{\mu\nu}$ [97]	$\sqrt{g}R\phi^2$	-	-	1	arb.	0
$\sqrt{g}, \sqrt{g}R$ [95]	$\sqrt{g}R\phi^2$	-	-	1	arb.	0
$\sqrt{g}, \sqrt{g}R$ [98]	$*\sqrt{g}R^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$	-	-	1	0	0
$\sqrt{g}, \sqrt{g}R$ [99]	$*\sqrt{g}R^{\mu\nu}\partial_\mu\phi\partial_\nu\phi * \sqrt{g}Rg^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$	- -	- -	1	0	0
$\sqrt{g}, \sqrt{g}R$ [96]	-	$\sqrt{g}R\bar{\psi}\psi$	-	0	arb.	0
$\sqrt{g}, \sqrt{g}R$ [91]	-	* $\sqrt{g}R^{\mu\nu}\bar{\psi}\gamma_\mu\nabla_\nu\psi$	-	0	arb.	0

Fermions in the Background-Field Approximation

背景场近似中的费米子

For fermions, there is a technical subtlety: if one chooses a regulator function which does not regularize the modes of the Dirac operator and works in the background-field approximation, the sign of a_F can be flipped to $a_F > 0$ [79, 86, 88, 93] .

对于费米子，存在一个技术难点: 如果选择的调节器函数不能正则化狄拉克算符的模式，并且在背景场近似下计算，a_F 的符号会翻转为 $a_F > 0$ [79, 86, 88, 93] 。

Impact of Standard Model Fields

标准模型场的影响

Synopsis: The four scalar components of the Higgs field and 45 Weyl fermions of the SM only partially counteract the antiscreening effect of gravitational modes and the 12 gauge fields; therefore, an asymptotically safe fixed point for the Newton coupling persists in presence of SM matter. The fixed point also exists when further gravitational couplings are included.

提要: 希格斯场的四个标量分量与标准模型的 45 种外尔费米子仅部分抵消引力模式和 12 种规范场的反屏蔽效应；因此，存在标准模型物质时，牛顿耦合仍然存在渐近安全不动点。纳入额外引力耦合后，该不动点依然存在。

It is an important observational consistency test for any model of quantum gravity, whether the observed matter fields of the SM can exist within the model. It is evidently not a given that asymptotically safe gravity passes this consistency test, given the results from the previous section ”Screening and Antiscreening Effects of Matter on the Newton Coupling”. First, passing the test requires that the screening effect of the scalars and fermions in the SM does not overwhelm the antiscreening effect of gravitational and gauge fields on the Newton coupling. Only then can an asymptotically safe fixed point for the Newton coupling exist. Second, passing the test requires that the fixed point continues to exist when further couplings beyond the Newton coupling are considered on the gravitational and the matter side. Matter couplings are discussed in section ”Global Symmetries Persist and Have Phenomenological Consequences”.

对任何量子引力模型而言，观测到的标准模型物质场能否容纳在模型中，都是一项重要的观测一致性检验。结合上一节“物质对牛顿耦合的屏蔽与反屏蔽效应”的结果来看，渐近安全引力能通过这项检验并非理所当然。首先，通过检验要求: 标准模型中标量和费米子的屏蔽效应不会压过引力场与规范场对牛顿耦合的反屏蔽效应。唯有如此，牛顿耦合的渐近安全不动点才能存在。其次，通过检验要求: 当考虑引力端和物质端牛顿耦合之外的其他耦合时，该不动点依然存在。物质耦合会在“全局对称性得以保留并具有现象学后果”一节中讨论。

All computations so far agree on the following important result: the asymptotically safe fixed point of pure gravity continues to exist, when the matter content of the SM is accounted for. The fixed-point values and critical exponents depend on the number of matter fields, but, if the number of matter fields of the three species (scalars, fermions, vectors) are treated as continuous parameters, the fixed-point values and critical exponents at $N_S = 0, N_F = 0, N_V = 0$ can continuously be connected to those at to $N_S = 4, N_F = 22.5, N_V = 12$ [45, 80, 86, 88, 102, 107] . Note that if a fixed point can be deformed continuously through increasing a continuous parameter from an initial to a final value, one can think of the final fixed point as a deformation of the original universality class. In contrast, if the fixed point at the final value of the parameter is not obtained through a deformation of the fixed point at the initial value, one cannot understand it as a deformation of the original universality class but instead has to think of these as two different universality classes. For this second case, the existence of the first universality class (at the initial value of the parameter) does not matter for the existence of the second universality class (at the final value of the parameter). Translated to gravity-matter systems, this second possibility would mean that a second asymptotically safe universality class is unrelated to the pure-gravity universality class. While this is a logical possibility, this does not appear to be the case.

迄今为止所有计算都得出了一致的重要结论: 纳入标准模型物质内容后，纯引力的渐近安全不动点仍然存在。不动点值和临界指数依赖于物质场的数量，但如果将三类物质 (标量、费米子、矢量) 的数量视为连续参数，$N_S = 0, N_F = 0, N_V = 0$ 处的不动点值和临界指数可以连续连接到 $N_S = 4, N_F = 22.5, N_V = 12$ [45, 80, 86, 88, 102, 107] 处的对应值。注意，如果不动点可以通过连续增大参数从初始值形变到终值，就可以将终态不动点视为原普适类的形变。反之，如果参数终值处的不动点不是初始值处不动点形变得到的，就不能将它理解为原普适类的形变，而必须将二者视为两个不同的普适类。对于第二种情况，第一个普适类 (参数初始值处) 的存在性与第二个普适类 (参数终值处) 的存在性无关。放到引力-物质系统中看，第二种可能性意味着第二个渐近安全普适类与纯引力普适类无关。尽管这在逻辑上是可能的，但实际情况似乎并非如此。

Therefore, asymptotically safe quantum gravity passes a crucial observational consistency test.

因此，渐近安全量子引力通过了一项关键的观测一致性检验。

Some extensions of the SM may also admit a fixed point. The most important extension is probably the addition of three right-handed neutrinos, which are required to explain neutrino oscillations (unless the neutrinos are Majorana). Further extensions may be necessary to accommodate dark matter, e.g., in the form of an axion or axion-like particle or a gauge-singlet scalar. Such extensions by 3 Weyl fermions (3/2 Dirac fermions) and one or two scalars are indeed possible in all studies to date.

部分标准模型扩展也可以存在不动点。最重要的扩展大概率是加入三个右手中微子——除非中微子是马约拉纳粒子，否则要解释中微子振荡就需要右手中微子。还可能需要额外扩展来容纳暗物质，例如轴子、类轴子粒子或规范单态标量形式的暗物质。迄今为止所有研究都表明，增加 3 个外尔费米子 (即 3/2 个狄拉克费米子) 和一到两个标量的这类扩展确实是可行的。

As a first step toward a supersymmetric setting, the inclusion of a gravitino, i.e., a spin 3/2 field, has been studied in [108].

作为迈向超对称框架的第一步，文献 [108] 已经研究了引力微子 (即自旋 3/2 场) 的纳入。

Preview on Phenomenological Consequences

对现象学后果的预告

There are indications for a mechanism that renders the SM plus gravity not only asymptotically safe but also more predictive than the SM on its own. This mechanism relies on matter fields changing gravitational fixed-point values. Here, we preview this mechanism and get back to it in more detail in section ”Toward a UV Completion of the Standard Model”. Yukawa interactions which the SM needs for its fermions to be massive, vanish, unless conditions on the gravitational fixed-point values are fulfilled [95, 109, 110]; see also section ”Toward a UV Completion of the Standard Model”. These conditions are not fulfilled by the gravitational fixed-point values, as they come out in studies with no or few matter fields [109-111]. However, once three generations of SM fermions are added, the gravitational fixed-point values satisfy the conditions according to the study in [80]; see Fig. 5. Thus, fermions push the gravitational fixed point into a region of values, in which fermion mass generation through Yukawa couplings to the Higgs field becomes possible.

有迹象表明，存在一种机制不仅能使标准模型加引力成为渐近安全的，还比单独的标准模型更具预言性。该机制依赖物质场改变引力不动点的值。在此，我们预先介绍该机制，之后会在“朝向标准模型的紫外完备”章节中展开详细讨论。除非引力不动点值满足特定条件，否则标准模中费米子获得质量所必需的汤川相互作用会消失 [95, 109, 110]，另见“朝向标准模型的紫外完备”章节。在无物质场或仅含少量物质场的研究中，得到的引力不动点值并不满足这些条件 [109-111]。然而，根据文献 [80] 的研究，一旦加入三代标准模型费米子，引力不动点值就会满足上述条件，参见图 5。因此，费米子将引力不动点推到了满足条件的取值区域，在该区域中，通过汤川耦合与希格斯场耦合产生费米子质量成为可能。

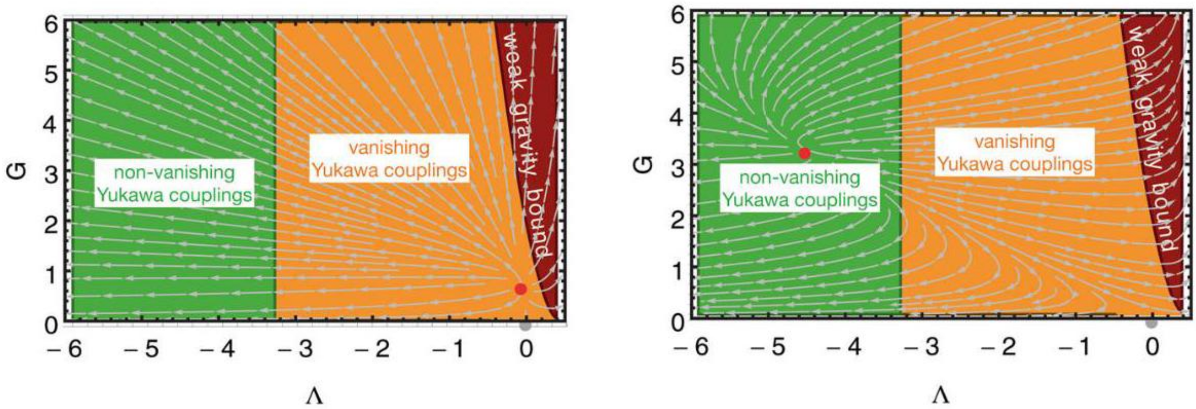


Fig. 5 We show the plane spanned by the dimensionless Newton coupling G_N and the dimensionless cosmological constant Λ . We also indicate the boundary between the region with nonvanishing Yukawa couplings (green) and the region with vanishing Yukawa couplings (orange). Additionally, the weak-gravity bound, see section ”Step 2: The Symmetry-Preserving Interactions Which Are Generated by Gravity Have an Asymptotically Safe Fixed Point”, is indicated. The fixed-point value (red dot) and the RG flow toward the IR in the background approximation are shown for one generation of Standard Model fermions (left panel) and three generations (right panel). In both panels, 12 vectors and four scalars are included. See [109] for the corresponding reference

图 5 我们展示了由无量纲牛顿耦合 G_N 和无量纲宇宙学常数 Λ 张成的平面。我们同时标出了非零汤川耦合区域 (绿色) 与零汤川耦合区域 (橙色) 的边界。此外，图中还标出了弱引力边界，相关介绍见“步骤 2: 引力生成的保对称性相互作用存在渐近安全不动点”章节。左图为一代标准模型费米子、右图为三代标准模型费米子的情况，两图均给出了不动点值 (红点) 和背景近似下流向红外的重整化群流。两个面板中都包含了 12 个矢量和四个标量。对应参考文献见 [109]

We caution that one needs to know the gravitational fixed-point values with relatively high accuracy to determine whether or not this mechanism is indeed at work. Current studies have not yet achieved the accuracy to comprehensively confirm the scenario in [109].

我们需要提醒的是，要确定该机制是否真的成立，需要以相当高的精度获知引力不动点的值。目前的研究尚未达到能全面证实 [109] 中这一方案的精度。

That the Yukawa couplings of the SM cannot be accommodated automatically is an important result in quantum gravity phenomenology, because it means that the asymptotically safe model is testable at SM scales: if the fixed-point value falls outside the green region in Fig. 5, the model is ruled out by an experimental result, namely, the measurement of nonvanishing Yukawa couplings at the LHC [112-117].

标准模型的汤川耦合无法自动满足这一点，是量子引力唯象学的一项重要结果，因为这意味着渐近安全模型可以在标准模型能标下被检验: 如果不动点值落在图 5 的绿色区域之外，那么该模型就会被一项实验结果排除——即 LHC 对非零汤川耦合的测量结果 [112-117]。

The Effective Strength of Gravity Under the Impact of Matter

物质影响下引力的有效强度

Synopsis: The effective strength of gravity can be encoded in a combination of couplings, in which these appear whenever gravitational fluctuations contribute to a system. This effective strength depends on the number of matter fields. Generically, scalar fields drive the effective strength up and fermions and vectors lower it.

概要: 引力的有效强度可编码为耦合组合，当引力涨落对系统有贡献时就会出现该组合。有效强度取决于物质场的数目: 一般而言，标量场会提高有效强度，费米子和矢量场会降低有效强度。

We introduce an effective strength of gravity, first discussed in [111]:

我们引入引力的有效强度，该概念最早在文献 [111] 中提出:

$$G_{\text{eff}} = \frac{G_N}{1-2\Lambda}. \tag{7}$$

Herein, Λ is the cosmological constant. G_{eff} is a combination of G_N and Λ , in which the two couplings enter in beta functions; therefore, this combination should have an asymptotically safe fixed point.

其中，Λ 是宇宙学常数。G_{eff} 是 G_N 与 Λ 的组合，这两个耦合出现在 β 函数中，因此该组合应当存在一个渐近安全不动点。

G_{eff} increases, when additional scalar fields are added to the system; see Fig. 6. Therefore, we expect that the system becomes increasingly nonperturbative, when scalars are added. Beyond a critical value of N_S , the asymptotically safe fixed point ceases to exist. We caution that the approximation in which studies are performed may break down at lower N_S than the critical value; see [82,89,92]. Nevertheless, a fixed point at small values of G_{eff} , where calculations are more easily controlled, may only exist at relatively small values of N_S .

往系统中添加额外标量场时，G_{eff} 会增大；参见图 6。因此我们预期，添加标量场会让系统的非微扰性越来越强。超过 N_S 的临界值后，渐近安全不动点将不复存在。我们提醒，相关研究采用的近似方法，可能在 N_S 低于临界值时就已经失效；参见 [82,89,92]。尽管如此，可更易控制计算的小 G_{eff} 取值处的不动点，可能仅存在于 N_S 相对较小的情况下。

For fermions, the situation differs, because G_{eff} decreases, when additional species are added [91]; see Fig. 6. In fact, the system may become increasingly perturbative, when fermions are added. However, to make a robust statement on this possibility, other couplings need to be analyzed as well. This may be important to connect asymptotically safe quantum gravity to the SM: it is known that the SM is perturbative at the Planck scale; thus, a UV completion with gravity has to be able to reproduce this perturbative regime. This is most likely achievable if the UV completion itself is (near-) perturbative in nature, cf. section ”On the Near-Perturbative Nature of Gravity-Matter Systems”. The SM has 22.5 Dirac fermions, which is sufficient to drive the effective gravitational strength to significantly lower values that for the pure-gravity case.

对于费米子，情况有所不同: 添加额外费米子时 G_{eff} 会降低 [91]；参见图 6。实际上，添加费米子可能让系统的微扰性越来越强。但要对该可能性给出可靠结论，还需要分析其他耦合。这一点对连接渐近安全量子引力与标准模型 (SM) 可能十分重要: 已知 SM 在普朗克能标下是微扰的，因此引力的紫外完备性必须能够重现这一微扰区域。若紫外完备性本身就是 (近) 微扰的，则该目标最有可能实现，参见章节“论引力-物质系统的近微扰性质”。SM 含有 22.5 个狄拉克费米子，这足以将有效引力强度降至远低于纯引力情形的数值。

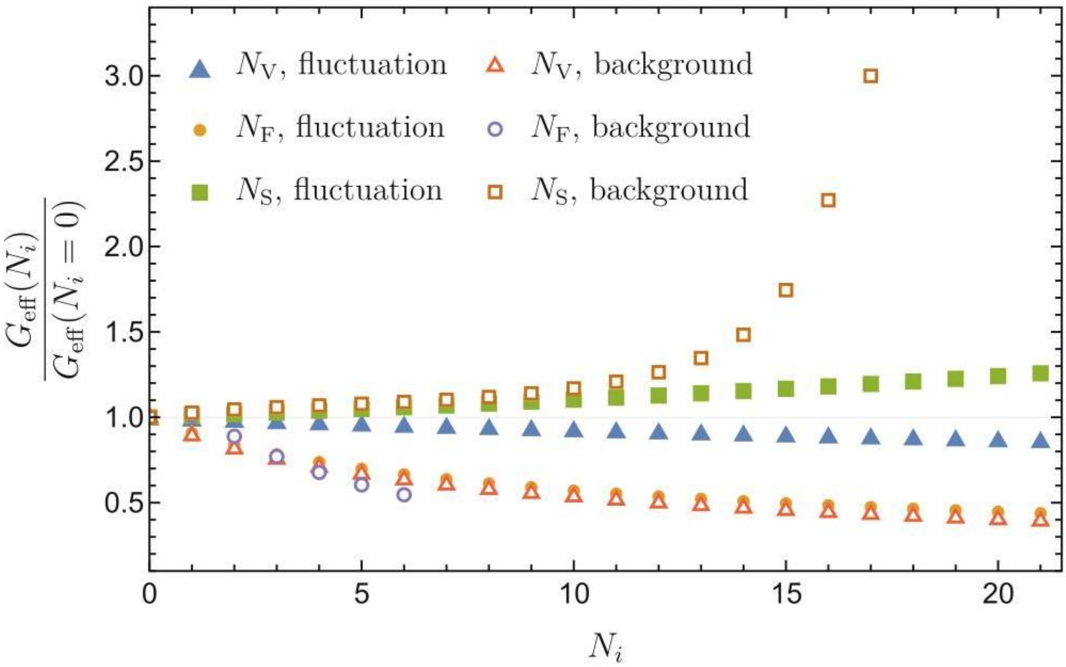


Fig. 6 We show the fixed-point values of the effective gravitational coupling G_{eff} (7) , when increasing the number of scalars, fermions, or vector fields for minimally coupled matter. The left column refers to fixed-point values obtained within fluctuation computations, taken from [82], and the right column refers to fixed-point values obtained within the background-field approximation, taken from [80]. We see that the qualitative behavior of G_{eff} agrees for all matter fields between the two different methods

图 6 我们展示了，对于最小耦合物质，增加标量场、费米子或矢量场数目时，有效引力耦合 G_{eff} (7) 的不动点取值。左列是涨落计算得到的不动点取值，取自 [82]，右列是背景场近似下得到的不动点取值，取自 [80]。可以看到，两种不同方法得到的所有物质场对应的 G_{eff} 定性行为一致。

G_{eff} decreases, when additional vectors fields are added to the system; see Fig. 6. Theories, which add additional gauge fields to the SM, may therefore be compatible with asymptotic safety. For Grand Unified Theories (GUT), the situation is not so clear, because they need additional scalars to spontaneously break the large gauge group. Different studies find differences on whether the matter content of popular GUT models is compatible with asymptotic safety, e.g., [80, 102], indicating that further studies are necessary.

往系统中添加额外矢量场时，G_{eff} 会降低；参见图 6。因此，给 SM 添加额外规范场的理论可能与渐近安全相容。对于大统一理论 (GUT)，情况尚不明确，因为这类理论需要额外标量场来自发破缺大规范群。不同研究对流行 GUT 模型的物质内容是否与渐近安全相容得出了不同结论，例如参见 [80, 102]，这表明仍需进一步研究。

Further Reading

延伸阅读

Integrating Out Matter Fields

积分积出物质场

In [87], the authors argue that truncations with minimally coupled matter fields are insufficient to infer whether or not bounds on the number of matter fields exist: they show that bounds disappear, if quantum fluctuations of matter fields are integrated out first, and of gravity last, instead of both simultaneously. If a truncation is large enough, the order in which fields are integrated out should be irrelevant.

在文献 [87] 中，作者认为带最小耦合物质场的截断不足以判断是否存在物质场数量的界限: 他们证明，如果先积分积出物质场的量子涨落、后积分积出引力的量子涨落，而非同时对两者积分积出，那么界限就会消失。如果截断足够大，积分积出场的顺序就应该不影响结果。

Additional Effective Gravitational Couplings

额外有效引力耦合

The definition in Eq. (7) can be generalized to

式 (7) 中的定义可以推广为

$$G_{\text{eff}}^{(n)} = \frac{G_{\text{N}}}{(1-2\Lambda)^n}. \tag{8}$$

For $n > 1$, these couplings also make an appearance in beta functions. For fermions, they have been compared in [91].

对于 $n > 1$ ，这些耦合也会出现在 β 函数中。对于费米子，二者已在文献 [91] 中进行了对比。

Comparing Background Approximation and Fluctuation Computations

背景近似与涨落计算对比

For fermions, the decrease of G_{eff} comes about in different ways depending on the choice of approximation: in [80], employing the background-field approximation, Λ becomes large and negative, which decreases G_{eff}, in [91], which constitutes a fluctuation computation, Λ stays approximately constant, while G_N decreases. Such differences between approximations do not matter at the level of G_{eff}, which is a more useful quantity to consider, both for its higher degree of robustness, and because G_{eff}, not G_N, enters beta functions and thus determines the strength of gravity fluctuations. More generally speaking, G_{eff} behaves qualitatively similar for computations employing the background-field approximation and for fluctuation computations for all three types of matter, i.e., scalars, fermions, and gauge fields; see Fig. 6.

对于费米子，G_{eff} 的降低方式会因近似选择的不同而不同: 在文献 [80] 中，采用背景场近似得到 Λ 为大负值，从而使 G_{eff} 降低；在构成涨落计算的文献 [91] 中，Λ 近似保持恒定，而 G_N 发生降低。这类近似之间的差异对 G_{eff} 层面并不产生影响，G_{eff} 是更适合纳入考虑的物理量，这既是因为它的鲁棒性程度更高，也因为进入 β 函数并决定引力涨落强度的是 G_{eff} 而非 G_N。更一般地说，对于所有三类物质场——即标量场、费米子场和规范场，采用背景场近似的计算和涨落计算中，G_{eff} 的定性行为是相似的；参见图 6。

Background-Field Approximation and Fluctuation Input

背景场近似与涨落输入

In the background-field approximation, the difference of background and fluctuation couplings is neglected; see section ”Methods to Investigate Asymptotically Safe

在背景场近似中，背景耦合与涨落耦合的差异被忽略；参见“研究渐近安全引力

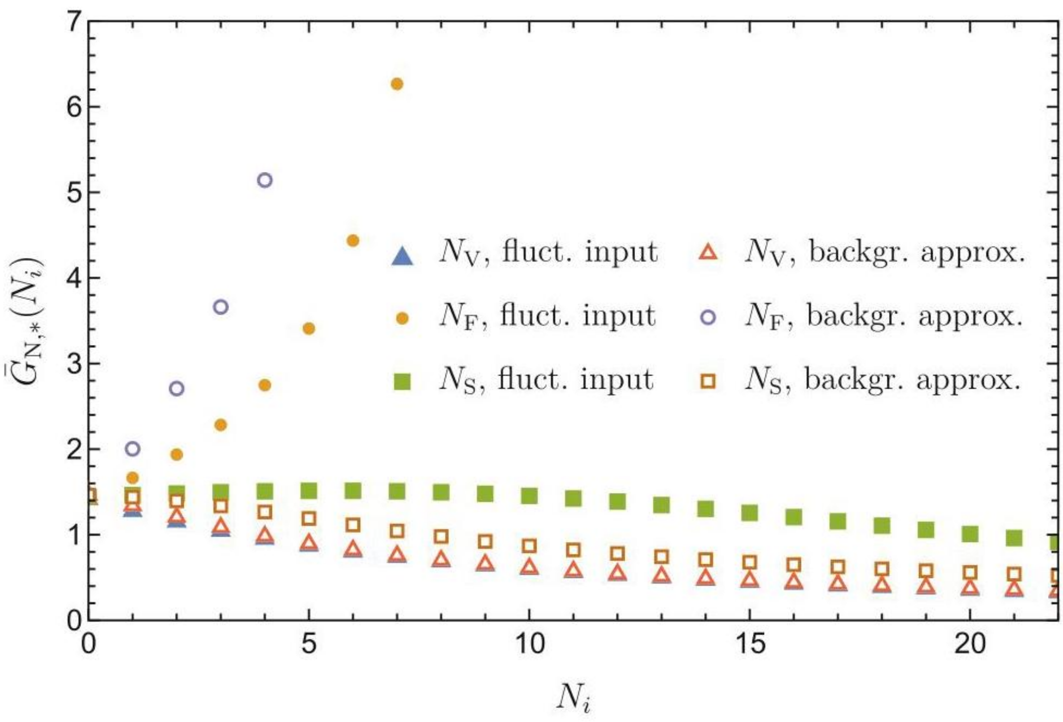


Fig. 7 We show the fixed-point values of the background Newton coupling $\bar{G}_N$ as a function of different matter fields. Full markers indicate the background-field approximation. Empty markers indicate an approximation, where the fluctuation couplings enter the right-hand side of the flow equation and therefore the beta functions of the background couplings

图 7 我们展示背景牛顿耦合 $\bar{G}_N$ 的不动点值，其为不同物质场的函数。实心标记代表背景场近似，空心标记代表将涨落耦合纳入流方程右手边，因而纳入背景耦合的贝塔函数的近似方案

Gravity-Matter Systems". A first step to lift this approximation can be achieved by taking into account the input of fluctuation couplings in the scale dependence of the background couplings. In this way, the fixed-point values for the fluctuation coupling enter the beta functions of the background couplings. In Fig. 7, we compare these two setups with each other. While the inclusion of fluctuation couplings changes the fixed-point values for $\bar{G}_N$ on a quantitative level, the qualitative features remain rather robust.

-物质系统"一节。要放宽这一近似，第一步可在背景耦合的标度依赖中纳入涨落耦合的贡献，此时涨落耦合的不动点值会进入背景耦合的贝塔函数。我们在图 7 中对比了这两种设置: 纳入涨落耦合后，$\bar{G}_N$ 的不动点值在定量层面发生改变，但定性特征仍相当稳健。

Global Symmetries Persist and Have Phenomenological Consequences

整体对称性存续并具有唯象学意义

Synopsis: Global symmetries, such as shift symmetry for scalar fields or chiral symmetries for fermions, are left intact by gravitational fluctuations. In turn, they determine through which terms matter fields interact at an asymptotically safe fixed point. These interactions may lead to a bound on the strength of gravity, the weak-gravity bound. Further, these interactions find a way to circumvent a mechanism for chiral symmetry breaking which would leave fermions with Planck-scale masses and would rule out asymptotic safety.

概要: 整体对称性，例如标量场的平移对称性或费米子的手征对称性，不会被引力涨破破坏。它们决定了物质场在渐近安全固定点处通过何种项发生相互作用。这些相互作用可能会给出引力强度的一个约束，即弱引力约束。此外，这些相互作用能够规避一种会使费米子获得普朗克能标质量并排除渐近安全的手征对称性破缺机制。

The Status of Global Symmetries in Asymptotic Safety

渐近安全中整体对称性的现状

Synopsis: There is a general argument suggesting that there cannot be global symmetries in quantum theories of gravity. We review the argument and point out its assumptions, which may not hold in asymptotically safe gravity. We then review explicit calculations that show that quantum fluctuations of gravity generate new interactions for matter fields. These interactions respect the maximum set of global symmetries of the corresponding matter fields. In contrast, interactions which violate the maximum set of global symmetries of the matter fields are not generated, i.e., can consistently be set to zero.

概要: 有一个普遍论点认为，量子引力理论中不可能存在整体对称性。我们回顾了该论点并指出了它的假设，这些假设在渐近安全引力中可能不成立。随后我们梳理了已有的显式计算，结果表明引力的量子涨落会为物质场生成新相互作用。这些相互作用保留了对应物质场的最大整体对称性集合。反之，破坏物质场最大整体对称性集合的相互作用并不会生成，即可以始终一致地设为零。

In quantum field theory, symmetries play a central role by dictating the interactions of the fields in the theory. Global symmetries play different roles - and are understood to varying degrees - at different scales. At condensed matter scales, many different global symmetries occur, under which order-parameter fields transform. The corresponding theories describe phases and phase transitions through the spontaneous breaking of these global symmetries. Moving toward smaller length scales, namely, those of particle physics, there is only a single global symmetry in the SM, namely, a global $U(1)_{\mathrm{B-L}}$, where B and L stand for baryon and lepton number. Additional global symmetries are present, if interactions are switched off. Beyond the SM, global symmetries play an important role, e.g., in dark-matter models, where they may ensure the stability of dark matter. Finally, moving toward even smaller scales, in the quantum-gravity regime, what is the fate of global symmetries?

在量子场论中，对称性通过决定理论中场的相互作用发挥核心作用。整体对称性在不同尺度发挥不同作用，人们对它的理解程度也各不相同。在凝聚态尺度，存在许多不同的整体对称性，序参量场在这些对称性下变换。对应的理论通过这些整体对称性的自发破缺描述相和相变。朝着更小的长度尺度，也就是粒子物理的尺度前进，标准模型中仅存在一个整体对称性，即整体 $U(1)_{\mathrm{B-L}}$ ，其中 B 和 L 分别代表重子数和轻子数。如果关闭相互作用，还会存在额外的整体对称性。超出标准模型的范围，整体对称性发挥重要作用，例如在暗物质模型中，它们可以保证暗物质的稳定性。最后，朝着更小的尺度前进，进入量子引力范畴，整体对称性的命运会是怎样的呢?

There is a general argument that states that any global symmetry should be broken in quantum gravity

[118, 119]; however, as any such arguments, it relies on assumptions which may or may not hold in a given setting. Note that in the context of string theory, this is known as the no global symmetries swampland conjecture; see, e.g., [120] and references therein. We will now present the argument and explain where assumptions are being made.

> 有一个普遍论点指出，任何整体对称性都应当在量子引力中破缺 [118, 119]；但和所有这类论点一样，它依赖的假设在给定情境下不一定成立。请注意，在弦论的语境中，这就是人们熟知的无整体对称性沼泽地猜想；例如参见 [120] 及其中的参考文献。我们接下来会呈现这个论点，并说明它在何处引入了假设。

The argument relies on virtual black-hole configurations in the gravitational path-integral, together with the assumption that global charges are not preserved by black holes. It says that among the various spacetime configurations that the gravitational path integral sums over, there are configurations that correspond to black holes. In turn, if one extrapolates Hawking radiation all the way to zero mass (i.e., through the semiclassical and also the quantum regime), then black holes destroy information on global charges: for instance, one can imagine building a black hole from only protons (so it has a well-defined baryon number and lepton number), but it evaporates not just into baryons, but into various elementary particles, completely destroying any memory of the initial global charge. Therefore, the argument concludes, there is a contribution in the gravitational path integral that destroys global charges, and thus, global symmetries cannot be preserved in quantum gravity.

> 该论点依赖引力路径积分中的虚黑洞构型，同时假设整体电荷不被黑洞保留。它指出，在引力路径积分求和的所有时空构型中，存在对应黑洞的构型。进一步来说，如果将霍金辐射外推一直到零质量 (即贯穿半经典区域乃至量子区域)，那么黑洞会摧毁整体电荷的信息: 例如，可以想象仅由质子构成黑洞 (因此它有明确的重子数和轻子数)，但它蒸发产生的不仅有重子，还有多种基本粒子，初始整体电荷的所有信息会被完全摧毁。因此该论点得出结论: 引力路径积分中存在破坏整体电荷的贡献，因此整体对称性无法在量子引力中保留。

However, first, the actual contribution of virtual black-hole configurations to the gravitational path integral is not known (and indeed depends on the microscopic dynamics - so may be different in an asymptotically safe setting than, e.g., when using the Einstein action). One can imagine settings where the microscopic dynamics S is such that the phase factor e^{iS} , when evaluated on black-hole configurations, leads to destructive interference; see, e.g., [121]. Second, whether or not global charges are conserved also depends on whether it is true that there are no black-hole remnants, for which, indeed, there are counter-indications in asymptotic safety [122, 123]. If there are black-hole remnants, i.e., the Hawking evaporation process stops at a finite mass of the black hole, then the original information on global charges may be stored by the remnant.

> 但首先，虚黑洞构型对引力路径积分的实际贡献尚不清楚 (而且它确实依赖微观动力学——因此在渐近安全情境下，它可能和爱因斯坦引力等情形不同)。可以想象，在微观动力学 S 使得相位因子 e^{iS} 在黑洞构型上计算时产生相消干涉的情境下，结果会不同；例如参见 [121]。其次，整体电荷是否守恒也取决于是否不存在黑洞残余这一论断是否正确，而在渐近安全中确实有相反的指征 [122, 123]。如果存在黑洞残余，即霍金蒸发过程在黑洞达到有限质量时停止，那么初始整体电荷的信息就可以被残余储存下来。

In turn, there is a general argument against the existence of remnants [124], which itself relies on assumptions about the behavior of those remnants in scattering processes.

反过来，也存在一个反对残余存在的普遍论点 [124]，该论点本身依赖关于残余在散射过程中行为的假设。

In summary, one cannot in general conclude that global symmetries are broken in quantum gravity without knowing more about the specific properties of the theory.

总而言之，在不了解理论具体性质的情况下，不能笼统得出整体对称性一定会在量子引力中破缺的结论。

To settle the question whether or not global symmetries are conserved in asymptotically safe gravity or whether indeed only local symmetries may exist, it is necessary to calculate the gravitational effect on matter systems with global symmetries. Indeed, many such calculations have been performed, which we review below. As an upshot, none of the calculations indicates that global symmetries are broken by asymptotically safe quantum-gravity effects. One reason may be that gravity-matter systems may be near perturbative in asymptotic safety (see section ”On the Near-Perturbative Nature of Gravity-Matter Systems”), and thus, nonperturbative contributions in the path integral, which may break global symmetries, are negligible. An important caveat to this is that calculations are done in a Euclidean regime. There, analytic continuations of black-hole spacetimes exist but are physically quite distinct from black holes in a Lorentzian regime, because Lorentzian signature is necessary for the existence of causal relations and thus horizons. Thus, while no indications for global symmetry breaking exist in asymptotic safety to date, the question is not fully settled yet, and a different result may be found in Lorentzian signature. First studies of asymptotic safety in Lorentzian gravity exist, which yield a fixed point similar to the Euclidean one [29,31].

为了解决渐近安全引力中整体对称性是否守恒、抑或实际上仅存在定域对称性这一问题，必须计算引力对具有整体对称性的物质系统的影响。事实上，学界已经完成了许多此类计算，我们将在下文综述。结论是，目前没有计算表明整体对称性会被渐近安全量子引力效应破坏。一个可能的原因是，引力-物质系统在渐近安全中可能接近微扰 (参见“论引力-物质系统的近微扰性质”一节)，因此路径积分中可能破坏整体对称性的非微扰贡献可以忽略。对此的一个重要说明是，现有计算均在欧几里得区域完成。该区域中存在黑洞时空的解析延拓，但它在物理上与洛伦兹区域的黑洞差异显著，因为因果关系、进而视界的存在都要求洛伦兹号差。因此，尽管目前渐近安全中还没有出现整体对称性破缺的迹象，但该问题仍未完全解决，在洛伦兹号差下可能会得到不同的结果。目前已有对洛伦兹引力中渐近安全的初步研究，得到了与欧几里得情形相似的不动点 [29,31]。

There are two ways in which asymptotically safe gravity could reduce the global symmetries of a matter system: the first is by explicitly generating new interaction terms for matter with a lower degree of symmetry and the second is by preventing an asymptotically safe fixed point in the theory space with the maximum global symmetry. For clarity, we illustrate the two possibilities in Fig. 8.

渐近安全引力破坏物质系统整体对称性有两种方式: 第一种是显式生成对称性更低的新物质相互作用项，第二种是在具有最大整体对称性的理论空间中阻止渐近安全不动点存在。为清晰起见，我们在图 8 中演示了这两种可能性。

The simplest setting in which the conservation of global symmetries can be investigated in asymptotic safety through explicit calculations looks as follows: a noninteracting field is considered, i.e., only a kinetic term is specified. This minimal coupling to gravity suffices for quantum gravity fluctuations to generate interaction terms. For a given field, the kinetic term has a maximum set of global symmetries - e.g., for N_S real scalar fields, there is an $O(N_S)$ symmetry, in addition to N_S separate shift symmetries (for each of the scalar fields). Under the impact of quantum gravity fluctuations, the field does not remain noninteracting, i.e., quantum gravity fluctuations generate interactions. If the first possibility for symmetry breaking is realized, then gravitational fluctuations generate interactions which break the global symmetries explicitly. If the second possibility is realized, then no such symmetry-breaking interactions are generated, but there is also no fixed point among the symmetry-preserving interactions. In contrast, if quantum gravity does not break global symmetries, then the interactions which are generated share the symmetries of the kinetic term, and these interactions admit an asymptotically safe fixed point.

可以通过显式计算研究渐近安全中整体对称性守恒的最简单框架如下: 考虑一个无相互作用场，即仅指定动能项。这种与引力的最小耦合足以让量子引力涨落生成相互作用项。对于给定场，其动能项具有最大的整体对称群——例如，对于 N_S 个实标量场，存在 $O(N_S)$ 对称性，此外还有 N_S 个独立的平移对称性 (对应每个标量场)。在量子引力涨落的影响下，场不会保持无相互作用状态，即量子引力涨落会生成相互作用。如果第一种对称性破缺可能性成立，那么引力涨落生成的相互作用会显式破坏整体对称性。如果第二种可能性成立，那么不会生成此类破缺对称的相互作用，但在保对称相互作用中也不存在不动点。反之，如果量子引力不破坏整体对称性，那么生成的相互作用会继承动能项的对称性，且这些相互作用容许存在一个渐近安全不动点。

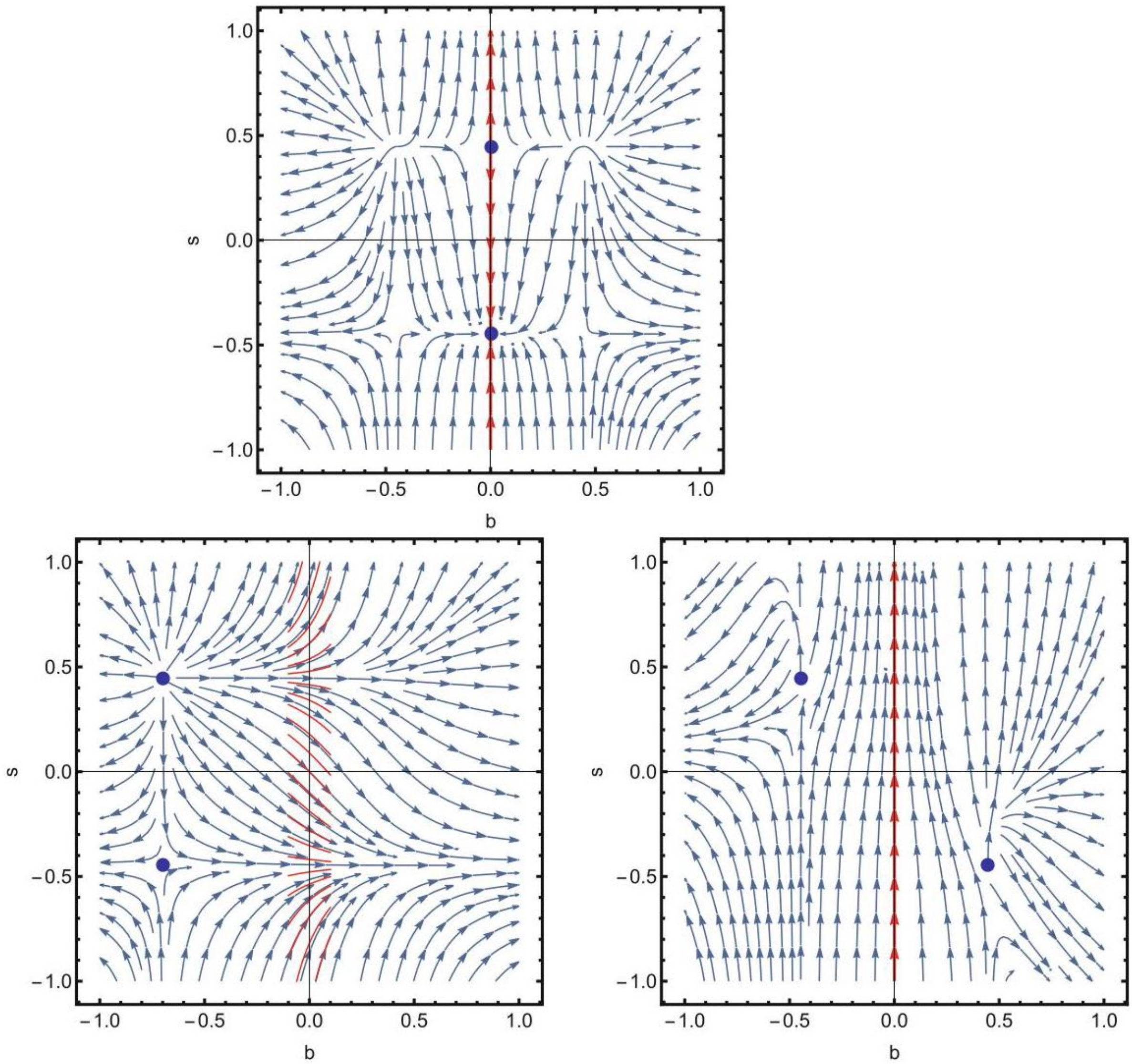


Fig. 8 We illustrate the three distinct possibilities for the status of global symmetries in asymptotically safe gravity. Here, s is a coupling that respects the global symmetry, whereas b breaks it. In the upper figure, the global symmetry is left intact, because quantum gravity does not generate the symmetry-breaking interaction: if $b = 0$ is chosen, the RG flow remains at $b = 0$ (red line); and because there are fixed points in the theory space with the global symmetry: they lie at $s \neq 0$ and $b = 0$ (blue dots). In both lower panels, the global symmetry is broken: in the left panel, gravity fluctuations induce b , even if it is set to zero (red lines). In the right panel, gravity fluctuations do not generate the symmetry-breaking interactions. However, there is no fixed point in the theory space with the global symmetry; the only fixed points lie at nonzero s and b

图 8 我们演示了渐近安全引力中整体对称性地位的三种不同可能性。此处，s 是遵守整体对称性的耦合，而 b 会破坏整体对称性。在上图中，整体对称性保持完整，因为量子引力不会生成破缺对称的相互作用: 如果选择 $b = 0$，重整化群流会保持在 $b = 0$ (红线)；同时，因为具有整体对称性的理论空间中存在不动点，它们位于 $s \neq 0$ 和 $b = 0$ (蓝点)。下方两个面板中整体对称性都被破坏: 左面板中，即使初始 b 为零，引力涨落也会诱导出 b (红线)。右面板中，引力涨落不会生成破缺对称的相互作用，但具有整体对称性的理论空间中不存在不动点；唯一的不动点位于非零的 s 和 b 处

In terms of the beta function for a coupling b that breaks a global symmetry, the question can be investigated as follows: the most general form of such a beta function is

对于破坏整体对称性的耦合 b 的 β 函数，我们可以按如下方式研究该问题: 这类 β 函数的最一般形式为

$$\beta_b = \beta_0 G_N^{\alpha_1} + \left(\beta_1 + \beta_1 G_N G_N^{\alpha_2}\right) b + \mathcal{O}\left(b^2\right). \tag{9}$$

If quantum gravity breaks the global symmetry through the generation of symmetry-breaking interactions, then $\beta_0 \neq 0$ must hold, so that any (partial or full) fixed point must have the symmetry-breaking interaction present.

如果量子引力通过生成破缺对称的相互作用破坏整体对称性，那么必须满足 $\beta_0 \neq 0$ ，因此任何 (部分或完全的) 不动点都必然存在破缺对称的相互作用。

Conversely, if $\beta_0 = 0$, gravity does not generate the symmetry-breaking interaction. Then, $b = 0$ is a zero of the beta function and the symmetry-breaking coupling can consistently be set to be zero, i.e., it is a partial fixed point for any value of G_N . Of course, this is not sufficient to guarantee that asymptotically safe gravity respects the corresponding global symmetry, because there may not be a full fixed point in the theory space with the global symmetry.

反过来，如果 $\beta_0 = 0$ ，引力不会产生破缺对称性的相互作用。此时 $b = 0$ 是贝塔函数的一个零点，破缺对称的耦合可以始终保持为零，也就是说，对任意 G_N 的取值，这都是一个部分固定点。当然，这并不足以保证渐近安全引力满足相应的整体对称性，因为理论空间中可能不存在满足该整体对称性的完整固定点。

To show that asymptotically safe gravity respects global symmetries, one therefore has to do two things: first, one has to show that symmetry-breaking interactions are not generated ($\beta_0 = 0$ in all corresponding beta functions). This means that the symmetry-breaking interactions have a partial fixed point at vanishing coupling values. Second, one has to show that the theory space with the maximum symmetry contains an asymptotically safe fixed point. This means that the partial fixed point extends to a full fixed point.

因此，要证明渐近安全引力满足整体对称性，必须完成两项工作: 首先，必须证明破缺对称的相互作用不会生成 ($\beta_0 = 0$ 在所有对应的贝塔函数中)。这意味着破缺对称相互作用在耦合为零处存在一个部分固定点。其次，必须证明具有最大对称性的理论空间包含一个渐近安全固定点，也就是说该部分固定点可以延拓为完整固定点。

Step 1: No Symmetry-Breaking Interactions Are Generated by Gravitational Fluctuations

第一步: 引力涨落不会产生破缺对称性的相互作用

Synopsis: In all examples with global continuous symmetries for matter fields which have been studied, these symmetries are preserved by quantum gravitational fluctuations, under the assumptions spelled out in the corresponding papers, which include Euclidean signature.

摘要: 在对应论文阐明的假设 (包括欧几里得号差) 下，所有已研究的物质场整体连续对称性案例中，这些对称性都能被量子引力涨落保留。

Continuous global symmetries that have explicitly been investigated include:

已被明确研究的整体连续对称性包括:

- For scalar fields:

  - 对于标量场:

- O (N_S) symmetry, under which an N_S -component scalar field transforms in the fundamental representation; $\phi^a \to O^{ab}_{N_S}\phi_b$, where $O^{ab}_{N_S}$ is in the fundamental representation. This symmetry would be broken, for example, if gravity generated distinct anomalous dimensions for some of the N_S scalars, or if gravity generated interaction terms with uneven numbers of scalars field, or if it generated different masses or interaction terms for some of the N_S scalars. Neither of these possibilities is realized in [83, 125].

  - O (N_S) 对称性，一个 N_S 分量标量场在该对称性下按基础表示变换；$\phi^a \to O^{ab}_{N_S}\phi_b$ ，其中 $O^{ab}_{N_S}$ 属于基础表示。若引力给部分 N_S 个标量生成不同的反常维数，或生成含不等量标量场的相互作用项，或给部分 N_S 个标量生成不同质量或相互作用项，该对称性就会被破缺。文献 [83, 125] 中证明这些情况都不会发生。

- Shift symmetry, under which $\phi \to \phi +$ const. This symmetry would be broken by a scalar potential. It was found in [78, 81, 126] that a scalar potential is not generated at the asymptotically safe fixed point. In contrast, it was found in [98,99,125,127,128] that shift-symmetric interactions (which are proportional to derivatives of the scalar field) are induced at an asymptotically safe fixed point.

  - 平移对称性，满足 $\phi \to \phi +$ 常数。该对称性会被标量势破缺。[78, 81, 126] 中的研究表明，渐近安全固定点不会生成标量势。与之相对，文献 [98,99,125,127,128] 发现，平移对称相互作用 (与标量场的导数成正比) 会在渐近安全固定点被诱导产生。

- A complex scalar, which has a global U(1) symmetry, was studied in [129], where quantum gravity does not generate terms that would break the global U(1) symmetry to a discrete $\mathbb{Z}_n$ symmetry. Thus, we find that quantum gravity generates interaction terms for scalar matter, which respect the maximum set of symmetries of the kinetic term, irrespective of the number of scalar fields. In addition, these interactions have a fixed point, if the weak-gravity bound is respected; see section ”Step 2: The Symmetry-Preserving Interactions Which Are Generated by Gravity Have an Asymptotically Safe Fixed Point” below.

文献 [129] 研究了具有整体 U(1) 对称性的复标量，该研究中量子引力不会产生能将整体 U(1) 对称性破缺为离散 $\mathbb{Z}_n$ 对称性的项。因此我们发现，无论标量场的数量是多少，量子引力给标量物质生成的相互作用项都满足动能项的最大对称集合。此外，只要满足弱引力边界，这些相互作用就存在固定点；参见下文“第二步: 引力生成的保对称相互作用存在渐近安全固定点”章节。

- For fermion fields:

- 对于费米子场:

- $SU(N_\mathrm{F})_L \otimes SU(N_\mathrm{F})_R$ symmetry, under which the left- and the right-handed components of N_F Dirac fermions transform separately. This is called a chiral symmetry, because it refers to the chiral components (the left- and right-handed Weyl spinors) of a Dirac fermion. This is a symmetry of the kinetic term, because a kinetic term for N_F Dirac fermions decomposes into a kinetic term for N_F right-handed and N_F left-handed Weyl spinors, $\bar{\psi}^i \mathcal{V}\psi^i = \bar{\psi}^i_L \mathcal{V}\psi^i_L + \bar{\psi}^i_R \mathcal{V}\psi^i_R$ (where $i = 1, \dots, N_\mathrm{F}$). This symmetry is broken by a mass term, $m\bar{\psi}^i \psi^i = m\bar{\psi}^i_R \psi^i_L + m\bar{\psi}^i_L \psi^i_R$, a nonminimal term of the form $R\bar{\psi}^i \psi^i = R\bar{\psi}^i_R \psi^i_L + R\bar{\psi}^i_L \psi^i_R$, a four-fermion interaction of the form $\bar{\psi}^i \psi^i \bar{\psi}^j \psi^j$, and others. Neither of these interactions is generated in the studies in [96, 111, 130, 131] . In contrast, it was found in [91, 96, 111, 130] that chirally symmetric four-fermion and nonminimal interactions are generated and have an asymptotically safe fixed point. Note that a breaking of chiral symmetry can be introduced explicitly by choosing a regulator that breaks chiral symmetry [93]; then, such chiral-symmetry-breaking interactions are generated.

- $SU(N_\mathrm{F})_L \otimes SU(N_\mathrm{F})_R$ 对称性，在该对称性下 N_F 狄拉克费米子的左手和右手分量分别变换。这被称为手征对称性，因为它指向狄拉克费米子的手征分量 (左手和右手外尔旋量)。这是动能项的对称性，因为 N_F 狄拉克费米子的动能项可以分解为 N_F 右手外尔旋量和 N_F 左手外尔旋量的动能项，即 $\bar{\psi}^i \mathcal{V}\psi^i = \bar{\psi}^i_L \mathcal{V}\psi^i_L + \bar{\psi}^i_R \mathcal{V}\psi^i_R$ (其中 $i = 1, \dots, N_\mathrm{F}$)。该对称性会被质量项 $m\bar{\psi}^i \psi^i = m\bar{\psi}^i_R \psi^i_L + m\bar{\psi}^i_L \psi^i_R$ 、形式为 $R\bar{\psi}^i \psi^i = R\bar{\psi}^i_R \psi^i_L + R\bar{\psi}^i_L \psi^i_R$ 的非最小项、形式为 $\bar{\psi}^i \psi^i \bar{\psi}^j \psi^j$ 的四费米子相互作用等破坏。在 [96, 111, 130, 131] 的研究中，这些相互作用都没有生成。相反，在 [91, 96, 111, 130] 中发现，手征对称的四费米子相互作用和非最小相互作用会被生成，且存在渐近安全固定点。请注意，如果选择破坏手征对称性的调节器，就可以显式引入手征对称性破缺 [93]；此时，这类破坏手征对称性的相互作用就会被生成。

Thus, we find that quantum gravity generates interaction terms for fermionic matter, which respect the maximum set of symmetries of the kinetic term, irrespective of the number of fermion fields. In addition, these interactions feature a fixed point under the inclusion of quantum gravity, without additional conditions; see section ”Light Fermions” below. The phenomenological consequences of this result entail that fermions can stay light in the presence of quantum gravity, as we will discuss below.

因此我们发现，量子引力为费米子物质生成的相互作用项满足动能项的最大对称集合，与费米子场的数量无关。此外，纳入量子引力后，这些相互作用天然存在固定点，无需额外条件；参见下文“轻费米子”一节。该结果的现象学意义在于，存在量子引力时费米子仍可以保持轻质量，我们将在下文讨论。

- For gauge fields:

- 对于规范场:

- O (N_V) symmetry, under which an N_V -component gauge field transforms in the fundamental representation; $A^a_\mu \to O^{ab}_{N_\mathrm{V}} A^b_\mu$, where $O^{ab}_{N_\mathrm{V}}$ is in the fundamental representation. Similar to the case for scalar fields, this symmetry is broken if gravitational fluctuations induce distinct anomalous dimensions for some of the gauge fields or interactions that only involve some of the N_V gauge fields. These possibilities are not realized [132], indicating that gravitational interactions do not break the global $O(N_V)$ symmetry.

- O (N_V) 对称性，在该对称性下一个 N_V 分量规范场按基础表示变换；即 $A^a_\mu \to O^{ab}_{N_V} A^b_\mu$，其中 $O^{ab}_{N_V}$ 属于基础表示。与标量场的情况类似，如果引力涨落为部分规范场诱导出不同的反常维数，或者生成仅涉及部分 N_V 规范场的相互作用，该对称性就会被破坏。这些可能性都没有实现 [132]，表明引力相互作用不会破坏整体 $O(N_V)$ 对称性。

- Shift symmetry in a gauge field is nothing but the global part of the Abelian gauge symmetry, which is of course also preserved by gravitational fluctuations.

- 规范场的移位对称性正是阿贝尔规范对称性的整体部分，当然也会被引力涨落保留。

Thus, we find that quantum gravity generates interaction terms for vector fields, which respect the maximum set of symmetries of the kinetic term, irrespective of the number of vector fields. In addition, these interactions feature a fixed point, if the weak-gravity bound is respected; see section "Step 2: The

因此我们发现，量子引力为矢量场生成的相互作用项满足动能项的最大对称集合，与矢量场的数量无关。此外，只要满足弱引力边界，这些相互作用就存在固定点；参见章节“第一步:

Symmetry-Preserving Interactions Which Are Generated by Gravity Have an Asymptotically Safe Fixed Point" below.

引力生成的保持对称性的相互作用存在渐近安全固定点”。

Step 2: The Symmetry-Preserving Interactions Which Are Generated by Gravity Have an Asymptotically Safe Fixed Point

第二步: 引力产生的保对称相互作用存在渐近安全固定点

Here, we proceed in two steps. We first explore, which interactions are generated by gravity. Second, we explore under which conditions these interactions have an asymptotically safe fixed point. If these conditions are fulfilled by the gravitational fixed-point values, then, together with step 1, the result suggests that global symmetries do indeed remain intact in asymptotically safe gravity-matter systems (with the caveats discussed above).

在此我们分两步开展研究: 首先探究引力会产生哪些相互作用，其次探究这些相互作用存在渐近安全固定点的条件。如果引力固定点取值满足这些条件，那么结合第一步的结果可知，整体对称性确实会在渐近安全引力-物质系统中保持不变 (存在上文讨论的注意事项)。

Step 2 a: Gravity Generates New Interactions for Matter

步骤 2a: 引力为物质生成新相互作用

Synopsis: There are interactions for matter which are necessarily generated by gravity, i.e., which cannot be set to zero consistently at an asymptotically safe fixed point. These interactions satisfy the symmetries of the kinetic term.

概要: 物质的部分相互作用必然由引力生成，也就是说，这些相互作用在渐近安全固定点上无法一致地设为零。这些相互作用满足动能项的对称性。

At an asymptotically safe fixed point, gravity is interacting. This implies that matter must also have interactions: because gravity couples to any form of energy and matter, it couples to any two free fields and generates an interaction between them - already classically. At the quantum level, the same statement is true, i.e., gravity induces interactions also at the loop level. The only way to switch off these induced interactions is to turn off the gravitational coupling, G_{N} . This is not possible when gravity is asymptotically safe; thus, asymptotically safe gravity-matter systems necessarily contain interactions for matter.

在渐近安全固定点，引力是存在相互作用的。这意味着物质也必须拥有相互作用: 因为引力耦合任意形式的能量与物质，它会耦合任意两个自由场，早在经典层面就会在二者之间生成相互作用。在量子层面，同样的结论依然成立，即引力在圈图层面也会诱导相互作用。关闭这些诱导相互作用的唯一方法是关掉引力耦合 G_{N} 。这在引力渐近安全的情况下是不可能的；因此，渐近安全引力-物质系统必然包含物质相互作用。

To see this explicitly, let us start with a scalar field ϕ which is minimally coupled to gravity and does not have any interactions. The minimal coupling is encoded in the kinetic term of the scalar field:

为了直观说明这一点，我们从一个最小耦合引力且不存在任何相互作用的标量场 ϕ 开始讨论。最小耦合编码在标量场的动能项中:

$$\Gamma_{k\,\mathrm{scal}} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi. \tag{10}$$

At the classical level, the presence of the metric in this kinetic term gives rise to gravity-mediated scattering via a tree-level diagram. At the quantum level, loop diagrams generate interaction terms for matter. In particular, the minimal coupling between the scalar field and gravity gives rise to scalar-gravity vertices, by expanding the kinetic term of the scalar in terms of metric fluctuations. Since we started from the kinetic term, the only coupling appearing in these vertices is the Newton coupling G_{N} . We use these vertices in one-loop diagrams with four external scalar fields and a loop of gravitational fluctuations as well as diagrams with gravitational fluctuations and scalars in the loop. Such diagrams generate a scalar self-interaction g_1 :

经典层面，该动能项中度规的存在会通过树图产生引力介导的散射。量子层面，圈图会生成物质的相互作用项。具体来说，通过对标量动能项按度规涨落展开，标量场与引力的最小耦合会生成标量-引力顶点。由于我们初始只有动能项，这些顶点中仅有的耦合就是牛顿耦合 G_{N} 。我们将这些顶点用于四类单圈图: 带有四个外标量场、一个引力涨落圈的图，以及带有引力涨落和标量在圈内的图。这类图会生成标量自相互作用 g_1 :

$$S_{\mathrm{Scal,\,int.}} = \frac{g_1}{8k^4}\int \mathrm{d}^4x\sqrt{g}g^{\mu\upsilon}g^{\rho\sigma}\partial_\mu\phi\partial_\upsilon\phi\partial_\rho\phi\partial_\sigma\phi. \tag{11}$$

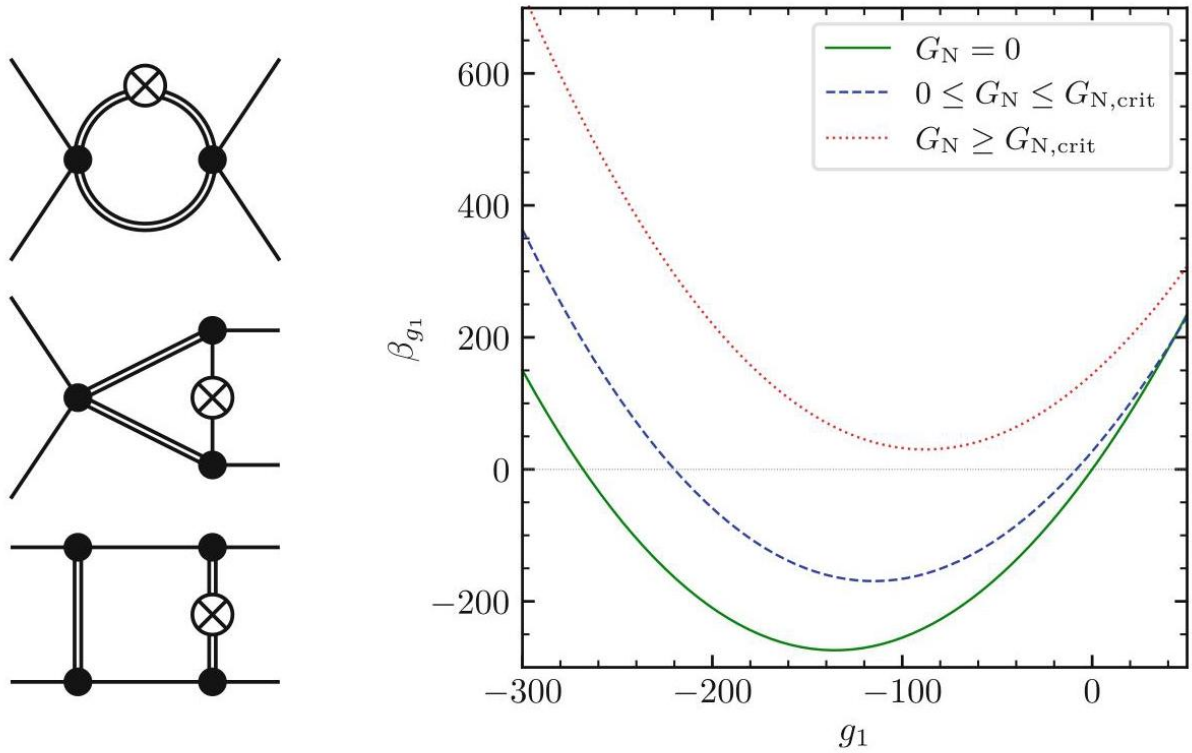


Fig. 9 Left panel: We show the diagrams through which gravitational fluctuations induce a nonvanishing fixed-point value for g_1 . The depicted diagrams contribute to the g_1 independent contribution C_0 of β_{g_1} ; see (12). The circle with cross indicates the regulator insertion $k\partial_k R_k$ of the flow equation. The depicted diagrams with the regulator insertion on all other internal lines, which are not shown separately, also contribute to C_0 . Right panel: We show the β -function for the induced coupling g_1 defined in (11) as a function of g_1 . For vanishing gravitational fluctuations (green solid line), $g_1 = 0$ is a fixed point, and g_1 can consistently be set to zero. For sufficiently small but nonzero values of the Newton coupling (blue dashed line), $g_1 = 0$ is not a (partial) fixed point anymore. Increasing the Newton coupling further, the two (partial) fixed points of β_{g_1} might collide, such that beyond a critical value of the Newton coupling β_{g_1} might not feature any fixed point anymore (red, dotted line); see section ”Step 2: The Symmetry-Preserving Interactions Which Are Generated by Gravity Have an Asymptotically Safe Fixed Point”

图 9 左图: 我们展示了引力涨落诱导出 g_1 非零固定点值的费曼图。所示图贡献了 β_{g_1} 中独立于 g_1 的项 C_0 ，参见 (12) 式。带叉的圆圈代表流方程的调节器插入 $k\partial_k R_k$ 。所示图 (调节器插在其他所有内线，这些内线没有单独画出) 也对 C_0 有贡献。右图: 我们展示了 (11) 式定义的诱导耦合 g_1 的 β 函数，它是 g_1 的函数。对于引力涨落为零的情况 (绿色实线)，$g_1 = 0$ 是一个固定点，g_1 可以一致设为零。对于足够小但非零的牛顿耦合 (蓝色虚线)，$g_1 = 0$ 不再是 (部分) 固定点。进一步增大牛顿耦合，β_{g_1} 的两个 (部分) 固定点可能碰撞，使得当牛顿耦合超过临界值后，β_{g_1} 可能不再存在任何固定点 (红色点线)；参见“步骤 2: 引力生成的保持对称性的相互作用存在渐近安全固定点”章节

We show the corresponding diagrams in the left panel of Fig. 9, stressing that all vertices are independent of g_1 . These diagrams contribute to the beta function of g_1 , which is schematically given by

我们在图 9 左面板展示了对应的费曼图，需要强调的是所有顶点都与 g_1 无关。这些费曼图对 g_1 的 β 函数有贡献，其形式可以概略写为

$$\beta_{g_1} = C_0 + C_1 g_1 + C_2 g_1^2 + \mathcal{O}\left(g_1^3\right), \tag{12}$$

where C_0 and C_1 are functions of the gravitational couplings. The coefficient C_0 contains the contribution of diagrams that do not contain a vertex with four scalar fields and come with G_N^2 , i.e., those in Fig. 9, such that $C_0 \to 0$ when $G_N \to 0$. Therefore, for vanishing gravitational fluctuations, i.e., $G_N = 0, g_1 = 0$ is a fixed point, cf. the blue line in Fig. 9. However, when gravitational fluctuations are present, the coefficient C_0 is non-vanishing, such that a (partial) fixed point for g_1 is necessarily nonzero, i.e., $g_{1,*}(G_N \neq 0) \neq 0$. Hence, in the presence of gravitational fluctuations, the coupling g_1 is necessarily induced, since it cannot be consistently set to zero. Instead, the (partial) Gaussian fixed point is shifted and becomes an interacting (partial) fixed point, which we will call shifted Gaussian fixed point (sGFP) in the following, cf. the green line in Fig. 9. Note that this is not a unique property of gravity: for a scalar field that is charged under an Abelian gauge field, a finite value of the gauge coupling also induces a four-scalar interaction which is similar to Eq. (11). In the case of charged matter, however, these interactions are induced under the RG flow toward the IR and not necessarily at the fixed point. Since the sGFP is continuously connected to the Gaussian fixed point, where g_1 is irrelevant, the induced coupling also corresponds to an irrelevant direction at the sGFP. Hence, gravitational fluctuations induce new interactions in the matter sector but do not necessarily introduce new relevant directions and hence do not reduce the predictivity of the theory.

其中 C_0 和 C_1 是引力耦合的函数。系数 C_0 包含不带有四标量场顶点的图的贡献, 这些图带有 G_N^2 , 即图 9 中的那些图, 因此当 $G_N \to 0$ 时满足 $C_0 \to 0$ 。因此, 对于引力涨落为零的情况, 即 $G_N = 0, g_1 = 0$ 是一个不动点，参见图 9 中的蓝线。但当存在引力涨落时，系数 C_0 非零，因此 g_1 的 (偏) 不动点必然非零，即 $g_{1,*}(G_N \neq 0) \neq 0$ 。因此，在存在引力涨落的情况下，耦合 g_1 必然被诱导产生，因为无法自洽地将其设为零。反之，(偏) 高斯不动点发生偏移，成为一个相互作用 (偏) 不动点，在下文中我们将其称为偏移高斯不动点 (sGFP)，参见图 9 中的绿线。请注意这并非引力独有的性质: 对于阿贝尔规范场下带电的标量场，有限的规范耦合也会诱导出类似式 (11) 的四标量相互作用。但在带电物质的情形下，这类相互作用是在朝向红外的重整化群流中被诱导产生的，并非一定出现在不动点处。由于 sGFP 与高斯不动点连续连通，而高斯不动点中 g_1 是不相关的，因此诱导耦合在 sGFP 处也对应一个不相关方向。因此，引力涨落会在物质区诱导出新相互作用，但不一定引入新的相关方向，因此不会降低理论的可预测性。

Crucially, the interaction in Eq. (11) respects the symmetries of the kinetic term, because it essentially corresponds to a square of the kinetic term. Similarly, one can show that higher-order induced interactions, or induced nonminimal interactions, satisfy the same symmetry requirement. This completes step 2a: we have reviewed that gravity generates matter interactions and those satisfy the global symmetries of the kinetic term.

关键之处在于，式 (11) 的相互作用满足动能项的对称性，因为它本质上对应动能项的平方。同理可以证明，高阶诱导相互作用或诱导非最小相互作用都满足相同的对称性要求。至此我们完成了步骤 2a: 我们已经说明引力会生成物质相互作用，且这些相互作用满足动能项的整体对称性。

While we introduced the induced coupling in the scalar sector, the same mechanism also takes place in the fermionic [130] and the gauge sector [133] and terms with a larger number of fields. In Table 2, we provide a list of induced interactions that have been explicitly studied in the literature. Generically, we expect all interactions that respect the symmetries of the kinetic terms, to be induced by gravitational fluctuations.

尽管我们是在标量区介绍了诱导耦合，但相同的机制也存在于费米子区 [130]、规范区 [133] 以及包含更多场的项中。在表 2 中，我们列出了文献中已经明确研究过的诱导相互作用。一般来说，我们预期所有满足动能项对称性的相互作用都会被引力涨落诱导产生。

Step 2 b: The Weak-Gravity Bound as a Condition Under Which a Symmetry-Preserving Fixed Point Exists

第二步 b: 弱引力边界是对称保持不动点存在的条件

Synopsis: It is nontrivial to satisfy the condition that all generated interactions from Step 2a have an asymptotically safe fixed point. While some of them have a partial fixed point for any value of gravitational couplings, others only have a partial fixed point (or a partial fixed point without new relevant directions) if gravity is sufficiently weakly coupled, i.e., if G_{eff} and its generalizations are sufficiently small. Such a weak-gravity bound (WGB) has been discovered for scalars, vectors, and for scalars coupled to fermions, although not for fermions on their own.

概要: 要满足 2a 步中所有生成相互作用都存在渐近安全不动点这一条件并非易事。其中部分相互作用对任意引力耦合值都存在部分不动点，另一部分仅当引力耦合足够弱时，也就是当 G_{eff} 及其推广形式足够小时，才存在部分不动点 (或不存在新增相关方向的部分不动点)。这类弱引力边界 (WGB) 已在标量场、矢量场以及与费米子耦合的标量场中被发现，尚未在单独存在的费米子中发现。

The WGB arises, because the beta function in Eq. (12) only has real zeros under certain conditions on the coefficients C_i . If we neglect contributions $\mathcal{O}\left(g_i^3\right)$, the sGFP is only real for

WGB 的出现原因是: 式 (12) 中的 β 函数仅在系数 C_i 满足特定条件时才存在实零点。若忽略贡献项 $\mathcal{O}\left(g_i^3\right)$ ，对称高斯不动点仅在下述条件下为实

$$4C_0C_2 \leq C_1^2 \tag{13}$$

For some systems, such as four-fermion couplings, this condition is satisfied automatically; see section ”Light Fermions”. For others, such as four-scalar couplings, this condition is only fulfilled up to critical values of the gravitational couplings. In fact, if gravity is too strongly coupled, i.e., simply put, G_{eff} (the strength of metric fluctuations) is too large, then the condition is violated. Therefore, the corresponding bound on the couplings is called the weak-gravity bound. There is also a system in the literature which exhibit a related bound [135]. In this system, a real fixed point exists for all G_{eff} , but it is characterized by additional relevant directions beyond a critical value of G . Such a change in universality class is undesirable, and thus, the gravitational coupling strength is also constrained; see below for more details. Note that there are two reasons for a change in universality class being undesirable. First, each additional relevant direction introduces a free parameter and thus limits the predictivity. Second, the interactions which become relevant are higher-order interactions, which, according to the most recent data from particle physics experiments, are suppressed at low energies, just as one would expect based on their canonical dimension. If their scaling dimension is changed by gravity, so that they are relevant, the suppression in the IR is no longer an automatic consequence of the RG flow; instead, larger values in the IR are also possible and change the phenomenology.

对于四费米子耦合这类系统，该条件会自动满足，参见“轻费米子”章节。对于四标量耦合这类其他系统，该条件仅在引力耦合达到临界值之前成立。实际上，若引力耦合过强，简单来说就是 G_{eff} (度规涨落的强度) 过大，该条件就会被破坏。因此，这一耦合对应的约束被称为弱引力边界。已有文献记载了一个存在同类约束的系统 [135]，该系统对任意 G_{eff} 都存在实不动点，但当 G 超过临界值后，该不动点会出现额外的相关方向。这种普适类的改变并不理想，因此引力耦合强度同样受到约束，更多细节参见下文。需要注意，普适类改变不理想有两点原因: 第一，每多一个相关方向就会引入一个自由参数，从而限制理论的预言能力；第二，变成相关的相互作用都是高阶相互作用，根据粒子物理实验的最新数据，这类相互作用在低能下是被压低的，这与从经典维数出发得到的预期一致。如果引力改变了它们的标度维数，使其变为相关相互作用，那么红外区的压低就不再是重整化群流的自然结果；相反，红外区也可能出现更大的耦合值，从而改变唯象结果。

Table 2 We list the interactions divided by $\sqrt{g}$ (and corresponding references) that were explicitly shown to be generated by quantum gravity. They all satisfy continuous global symmetries which are the maximum continuous global symmetries of their respective kinetic terms

表 2 我们按 $\sqrt{g}$ 分类列出已被明确证明由量子引力生成的相互作用 (及对应参考文献)。所有这些相互作用都满足连续整体对称性，该对称性正是它们各自动能项的最大连续整体对称性

Field	Global symmetry	Self-interaction	Nonminimal inter.	Ref.
Single scalar	Shift	$(\partial_\mu\phi\partial^\mu\phi)^2$	-	[127]
Single scalar	Shift	-	$\partial_\mu\phi\partial_\nu\phi R^{\mu\nu}$	[98]
Single scalar	Shift	$(\partial_\mu\phi\partial^\mu\phi)^2$	$\partial_\mu\phi\partial_\nu\phi R^{\mu\nu}$ & $\partial_\mu\phi\partial^\mu\phi R$	[99, 100]
N_S scalars	N_S shift symmetries & $O(N_S)$ symmetry	$\partial_\mu\phi^a\partial^\mu\phi^a\partial_\nu\phi^b\partial^\nu\phi^b$ $\partial_\mu\phi^a\partial^\mu\phi^b\partial_\nu\phi^a\partial^\nu\phi^b$	- -	[125]
Single vector	Shift	$(F_{\mu\nu}F^{\mu\nu})^2$	-	[133, 134]
Single vector	Shift	$(F_{\mu\nu}F^{\mu\nu})^2$ &$(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$	- -	[132]
N_V vectors	Shift	$(F^a_{\mu\nu}F^{a\mu\nu})^2$	-	
	& $O(N_V)$ symmetry	&$(F^a_{\mu\nu}\tilde{F}^{b\mu\nu})^2$	-	[132]
N_{F} fermions	Chiral	$(\bar{\psi}^i\gamma_\mu\psi^i)(\bar{\psi}^j\gamma^\mu\psi^j)$ & $(\bar{\psi}^i\gamma_\mu\gamma_5\psi^i)(\bar{\psi}^j\gamma^\mu\gamma_5\psi^j)$	- -	[85, 130, 131]
N_{F} fermions	Chiral	-	$R^{\mu\nu}\bar{\psi}^i\gamma_\mu\nabla_\nu\psi^i$	[91]
Single scalar and single fermion	Shift and chiral	$(\bar{\psi}\gamma^\mu D_\nu\psi)(\partial_\mu\phi\partial^\nu\phi)$ & $(\bar{\psi}\not{D}\psi)(\partial_\nu\phi\partial^\nu\phi)$	- -	[110, 111]

When studying induced interactions and the WGB, one usually proceeds order by order in canonical dimension. All induced interactions are canonically irrelevant, because they are essentially the square, or higher powers, of the kinetic terms. For a single shift-symmetric scalar field, the scalar self-interaction (11) is the only self-interaction up to this level of the canonical mass dimension. The coupling g_1 is indeed induced by gravitational fluctuations [99,125,127], since the coefficient C_0 in Eq. (12) is nonzero in general. This induced interaction gives rise to a WGB [125,127], which excludes a part of the plane spanned by the Newton coupling G_{N} and the cosmological constant Λ from the viable gravitational parameter space; see the left panel of Fig. 10. In G_N and Λ , it is less straightforward to see where the strong coupling regime is, because $\Lambda \to 1/2$ is also a strong coupling limit, not just $G_N \gg 1$. We therefore work in terms of the effective strength of metric fluctuations, G_{eff} and $G^{(2)}_{\text{eff}}$, defined in Eqs. (7) and (8), respectively. Both $G^{(2)}_{\text{eff}}$ and $G^{(2)}_{\text{eff}}$ can be thought of as measures of the strength of metric fluctuations and dominate the scale dependence of induced matter interactions. As we can see in the right panel of Fig. 10, the WGB is described by a rather constant value of

$G^{(2)}_{\rm eff}$, which enters the diagrams in Fig. 9 at leading order, for some range of Λ . Deviations from the constant appear due to dependencies on $G_{\rm eff}$.

研究诱导相互作用与弱引力边界 (WGB) 时，人们通常按正则维度逐阶分析。所有诱导相互作用在正则意义上都是不相关的，因为它们本质上是动能项的平方或更高次幂。对于单个平移对称标量场，在该正则质量维度水平下，标量自相互作用 (11) 是唯一的自相互作用。由于式 (12) 中的系数 C_0 通常不为零，耦合 g_1 确实由引力涨落诱导产生 [99,125,127]。这种诱导相互作用催生了弱引力边界 (WGB)[125,127]，它将牛顿耦合 $G_{\rm N}$ 与宇宙学常数 Λ 张成的平面的一部分排除出可行引力参数空间，参见图 10 左面板。在 G_N 和 Λ 中，强耦合区域的位置并不直观，因为不仅 $G_N \gg 1$ ，$\Lambda \to 1/2$ 本身也是强耦合极限。因此我们分别用式 (7) 和 (8) 定义的有效度规涨落强度 $G_{\rm eff}$ 和 $G^{(2)}_{\rm eff}$ 来开展分析。$G^{(2)}_{\rm eff}$ 和 $G^{(2)}_{\rm eff}$ 都可被视作度规涨落强度的度量，并且主导了诱导物质相互作用的标度依赖关系。从图 10 右面板可以看出，在 Λ 的一定范围内，WGB 对应 $G^{(2)}_{\rm eff}$ 近似为常数，该量领头阶进入图 9 的费曼图中。对常数的偏离来源于对 $G_{\rm eff}$ 的依赖。

When adding more scalar fields to the system, only those interactions that respect the $O(N_{\rm S})$ symmetry of the kinetic term are induced by gravitational fluctuations [125]. Increasing the number of scalar fields $N_{\rm S}$ makes the WGB stronger, such that more gravitational parameter space is excluded.

向系统中添加更多标量场时，只有满足动能项 $O(N_{\rm S})$ 对称性的相互作用才会被引力涨落诱导产生 [125]。增加标量场数量 $N_{\rm S}$ 会使 WGB 更严格，从而排除更多引力参数空间。

For fermions, the leading-order generated interactions are special, because they contain information not just about global symmetries. Because the maximum symmetry for fermions is chiral, i.e., distinguishes left- and right-handed fermions, the generated interactions also contain information about fermion masses. Thereby, they provide an important observational consistency test of asymptotic safety, because fermion masses in the SM are measured. We therefore devote section ”Light Fermions” to the discussion of generated fermion interactions and their phenomenological consequences. For the present section, it is only relevant that no WGB has been discovered yet in purely fermionic systems.

对于费米子，领头阶生成的相互作用有特殊性，因为它们不仅包含整体对称性的信息。由于费米子的最大对称性是手征对称性 (即区分左手和右手费米子)，生成的相互作用也包含费米子质量的信息。因此，它们为渐近安全提供了重要的观测一致性检验，因为粒子物理标准模型中的费米子质量已经被测量。我们因此将“轻费米子”一节专门用于讨论生成的费米子相互作用及其唯象学后果。就本节而言，仅需关注一点: 目前在纯费米子系统中尚未发现 WGB。

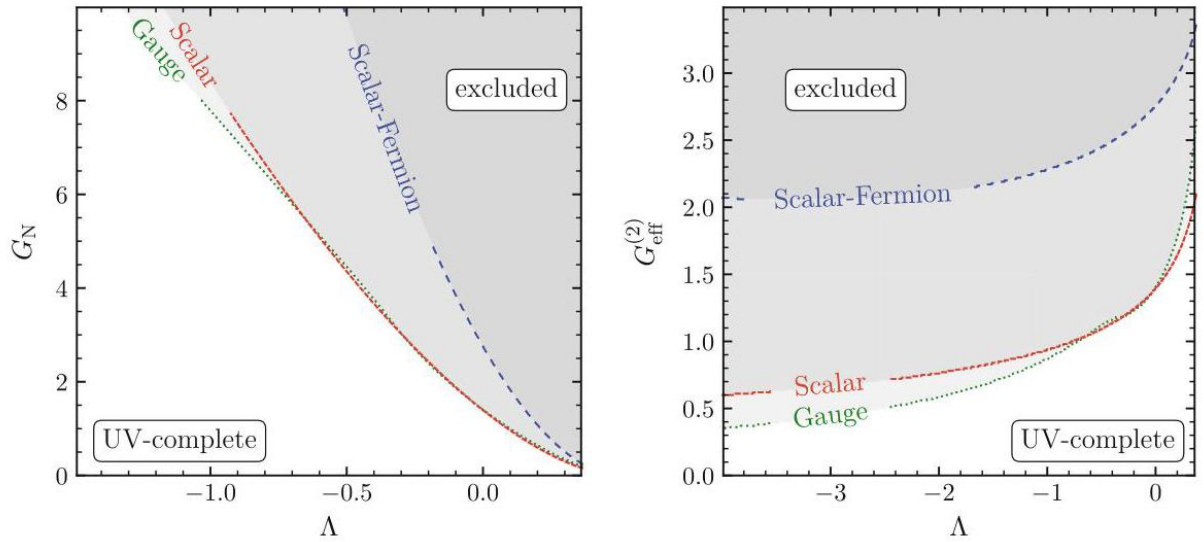


Fig. 10 We show the WGB for a shift symmetric scalar (see (11)), an Abelian gauge field (see (15)), and for a scalar-fermion system (14). In the gray region, no partial fixed point for the respective induced coupling exists, such that asymptotic safety cannot be realized in the respective region of the gravitational parameter space. It is therefore excluded from the viable parameter space. In the left panel, we show the WGB in the plane spanned by the gravitational coupling Λ and G_{N} , indicating that for any value of Λ , there is a critical value $G_{\mathrm{N,\,crit}}$ at which the asymptotically safe partial fixed point for the induced coupling vanishes. In the right panel, we show the same WGB, but in a plane spanned by Λ and the effective gravitational coupling $G_{\text{eff}}^{(2)}$, see (8)

图 10 我们展示了平移对称标量 (见 (11))、阿贝尔规范场 (见 (15)) 以及标量-费米子系统 (见 (14)) 的 WGB。在灰色区域中，对应诱导耦合不存在部分不动点，因此渐近安全无法在该引力参数空间区域实现，故该区域被排除出可行参数空间。左面板中，我们展示了引力耦合 Λ 与 G_{N} 张成平面内的 WGB，结果显示对于任意 Λ 值，都存在一个临界值 $G_{\mathrm{N,\,crit}}$ ，超过该值后诱导耦合的渐近安全部分不动点消失。右面板中，我们展示了同一个 WGB，但它处于 Λ 和有效引力耦合 $G_{\text{eff}}^{(2)}$ 张成的平面内，参见式 (8)

The situation is different for systems with fermions and scalars. The leading-order induced interactions are

费米子与标量混合系统的情况不同。领头阶的诱导相互作用为

$$S_{\text{Scal-Ferm, int.}} = \frac{i}{k^4}\int \mathrm{d}^4x \Big(\chi_1 \left[\bar{\psi}\gamma^\mu D_\upsilon \psi - \left(D_\upsilon \bar{\psi}\right)\gamma^\mu \psi\right]\partial_\mu\phi\partial^\upsilon\phi \tag{14}$$

$$+\chi_2 \left[\bar{\psi}\gamma^\mu D_\mu \psi - \left(D_\mu \bar{\psi}\right)\gamma^\mu \psi\right]\partial_\upsilon\phi\partial^\upsilon\phi\Big),$$

which indeed give rise to a WGB [110,111].

它确实催生了 WGB [110,111]。

For a single gauge field and at lowest order in the canonical mass dimension, there are two linearly independent induced four-vector interactions, namely,

对于单个规范场，在正则质量维度最低阶下，存在两个线性无关的诱导四矢量相互作用，即:

$$S_{\text{Vector, int.}} = \frac{1}{8k^4} \int \mathrm{d}^4 x \left(w_2 \left(F_{\mu\nu} F^{\mu\nu} \right)^2 + \kappa_2 \left(F_{\mu\nu} \widetilde{F}^{\mu\nu} \right)^2 \right), \tag{15}$$

with the field strength tensor F and the dual field strength $\widetilde{F}$. These interactions give rise to a WGB [132, 133], which generalizes to the case of several gauge fields [132].

其中 F 是场强张量，$\widetilde{F}$ 是对偶场强。这些相互作用催生了 WGB[132, 133]，该结论可以推广到多规范场的情况 [132]。

Comparing the WGBs from the different sectors - scalar, scalar-fermion, and gauge - we find a nearly universal curve; see Fig. 10. This indicates that the different sectors are equally sensitive to gravitational fluctuations.

对比来自不同区域——标量区、标量-费米子区和规范区——的弱引力边界，我们得到了一条近似普适的曲线；见图 10。这表明不同区域对引力涨落的敏感度是相同的。

Further Reading

扩展阅读

Preservation of Global Symmetries: Beyond Truncations

整体对称性的保留: 超越截断近似

As in any computation relying on the FRG, practical computations require choosing a truncation, i.e., only take a finite subset of interactions into account. All statements above on global symmetry breaking are therefore made within a truncation. However, from the structure of the flow equation, one can infer that the specific statements above, which pertain to the non-generation of symmetry-breaking interaction terms, generalize beyond truncations; see [136] for a discussion. Further, [99] contains a proof of shift-symmetry preservation in scalar-gravity systems.

和所有依赖泛函重整化群 (FRG) 的计算一样，实际计算需要选定截断，也就是仅将有限子集的相互作用纳入考量。因此上文所有关于整体对称性破缺的结论都是在截断近似下得到的。不过从流方程的结构可以推断，上文这些关于不会生成对称性破缺相互作用项的具体结论可以推广到截断近似之外；相关讨论参见文献 [136]。此外，文献 [99] 给出了标量-引力系统中平移对称性保留的证明。

WGB: Condition to Prevent Symmetry Breaking and Constraint on Gravitational Parameter Space

WGB: 防止对称性破缺的条件与引力参数空间的约束

There are two points of view on the WGB: the first is as a necessary condition to prevent the breaking of global symmetries; the second is as a condition on microscopic gravitational couplings that arises because matter-gravity theories should have fixed points. This second view has mostly been discussed in the literature; see [85, 91, 98-100, 110, 111, 125, 127, 130-134].

关于 WGB 有两种观点: 第一种是，它是防止整体对称性破缺的必要条件；第二种是，它是对微观引力耦合的约束，源于物质-引力理论应当存在不动点这一要求。文献中已对第二种观点进行了大量讨论，参见 [85, 91, 98-100, 110, 111, 125, 127, 130-134]。

The WGB and Asymptotically Safe Gravity-Matter Systems

WGB 与渐近安全引力-物质系统

Ultimately, we would like to know if scalar-gravity, or more generally, gravity-matter systems can be asymptotically safe with the maximum set of symmetries. This is the case if gravitational fixed-point values in the presence of matter satisfy the WGB. For systems with N_V Abelian gauge fields, this is the case in all studies to date. For scalars, the answer changes, when nonminimal interactions are accounted for: for N_S minimally coupled scalars, there is no fixed point of the full scalar-gravity systems, i.e., the WGB is violated [125]. At nonminimal coupling, the WGB holds for a single scalar [99, 100]. It is an open question whether this result may change under the inclusion of further interactions and whether the WGB is violated at $N_S > 1$.

归根结底，我们想要知道标量-引力，或更广泛而言，引力-物质系统能否在保留最大对称性集合的前提下实现渐近安全。只要存在物质时引力的不动点值满足 WGB，这一情况就能成立。对于包含 N_V 阿贝尔规范场的系统，迄今为止所有研究都表明这一条件成立。对于标量，当考虑非最小相互作用后答案会发生改变: 对于 N_S 最小耦合标量，完整的标量-引力系统不存在不动点，也就是 WGB 遭到违反 [125]。在非最小耦合情况下，单个标量满足 WGB[99, 100]。这一结果是否会在纳入更多相互作用后发生改变，以及 WGB 是否会在 $N_S > 1$ 处被违反，目前仍是开放性问题。

Robustness of the WGB

WGB 的鲁棒性

As a test of robustness, gauge parameter dependence of the WGB has been studied in [125] and in [132] and is weak in both cases. Eichhorn et al. [132] also demonstrates that it is important to include a full basis of generated interactions at leading order in canonical power counting. If not all interactions are included, the results strongly depend on the gauge on a qualitative level.

作为鲁棒性测试，文献 [125] 和 [132] 均研究了 WGB 的规范参数依赖性，结果表明两种情况下该依赖性都很弱。Eichhorn 等人 [132] 还证明，在正则幂计数领头阶中包含全部生成相互作用基是十分重要的。如果没有纳入所有相互作用，结果在定性层面会强烈依赖于规范选择。

Similarly, the exact location of the WGB depends on the chosen regulator function. It is important to emphasize that while the location of the WGB depends on the choice of regulator, so do the fixed-point values. Thus, a universal statement, namely, whether or not a fixed point exists, is expected to be regulator independent. However, no systematic investigations of this residual regulator dependence was conducted so far; see [135] for first steps.

类似地，WGB 的确切位置也依赖于所选的调节函数。需要强调的是: 不仅 WGB 的位置依赖于调节函数的选择，固定点值也同样如此。因此，“是否存在固定点”这一普适结论被认为与调节函数无关。但迄今为止，尚未对这种剩余调节依赖性开展系统研究；文献 [135] 是该方向的初步探索。

In [135], the WGB for a single scalar field has been investigated by considering a minimal coupling of gravity to a full function of the kinetic term for the scalar field. By studying the fixed-point structure of the pure-matter system upon expansion of the full function, it was concluded that only the free fixed point is a viable fixed point of the matter system; see also [137]. Accordingly, the WGB, which in this system arises as a collision between the (shifted) GFP and an interacting ”fixed point,” is interpreted as a truncation artifact. Nevertheless, the gravitational coupling strength is limited in this system, because the shifted Gaussian fixed point acquires new relevant directions if G is too large. Note that typically, when a fixed point acquires a new relevant direction, a fixed-point collision is involved. This can be seen, because to change sign, a critical exponent must pass through zero. In turn, a vanishing critical exponent indicates a degenerate zero of the beta function, i.e., two degenerate fixed points. In the scalar system in [135], no stable second fixed point exists; therefore, the change of sign proceeds in a different way: the leading critical exponent becomes complex and its real part switches sign, while its imaginary part is nonzero. In this way, the universality class changes without a fixed point collision.

在文献 [135] 中，研究者通过考虑引力与单个标量场动能项完整函数的最小耦合，研究了该单标量场体系的 WGB。通过对完整函数展开后研究纯物质体系的固定点结构，研究得出结论: 该物质体系仅自由固定点是可行固定点；参见文献 [137]。据此，该体系中由 (平移后的) 高斯固定点 (GFP) 与一个相互作用“固定点”碰撞产生的 WGB 被解释为截断赝象。尽管如此，该体系中引力耦合强度仍受到限制: 当 G 过大时，平移后的高斯固定点会获得新的相关方向。注意，通常当一个固定点获得新的相关方向时，会伴随发生固定点碰撞。这是因为，临界指数要改变符号必须经过零点。而临界指数为零意味着 β 函数存在简并零点，即两个简并固定点。在文献 [135] 的标量体系中，不存在稳定的第二个固定点，因此符号改变以另一种方式发生: 领头阶临界指数变为复数，其实部改变符号，而虚部保持非零。通过这种方式，普适类无需固定点碰撞即可发生改变。

Ultimately, both versions of the WGB are tied to the real part of a critical exponent becoming zero. In the usual version of the WGB, the fixed point is complex after the change in critical exponent, while the fixed point remains real in the novel version of the WGB. Both versions have in common that the shifted Gaussian fixed point with only irrelevant directions ceases to exist, if gravity is too strongly coupled.

归根结底，两种形式的 WGB 都与临界指数实部变为零相关。在常规 WGB 中，临界指数改变后固定点变为复数；而在新形式的 WGB 中，固定点保持为实数。两种形式的共同点是: 当引力耦合过强时，仅含无关方向的平移后高斯固定点将不复存在。

Light Fermions

轻费米子

Synopsis: All fermions in the SM are light compared to the Planck scale. This is a consequence of chiral symmetry, which prevents the generation of fermion masses and which is only broken spontaneously at the electroweak scale by the Higgs mechanism and below by QCD. In asymptotically safe quantum gravity, there are several conceivable mechanisms to break chiral symmetry, which would lead to inconsistencies with the observation of light fermions. Avoidance of some of these mechanisms puts lower and upper bounds on the number of light fermions. For the number of fermions in the SM, chiral symmetry is not broken in asymptotically safe quantum gravity.

摘要: 标准模型中的所有费米子都比普朗克尺度轻得多。这是手征对称性的结果: 手征对称性会阻止费米子质量产生，它仅在电弱尺度被希格斯机制自发破缺，在更低能标被量子色动力学破缺。在渐近安全量子引力中，存在多种可能的手征对称性破缺机制，这些机制会导致与轻费米子观测结果不一致。排除其中部分机制后，可以得到轻费米子数的上下界。对于标准模型中的费米子数，手征对称性不会在渐近安全量子引力中破缺。

The masses of the fermions in the SM range from several hundred keV to several GeV; hence, they are very light compared to the Planck scale. The reason for this is that chiral symmetry in the SM is only broken spontaneously at the electroweak scale. A chiral symmetry allows to rotate left- and right-handed fermions ψ_L and ψ_R independently, where the left- and right-handed component of a Dirac fermion can be extracted through the projection operators $P_{R/L} = \frac{1}{2}(1 \pm \gamma_5)$ as $\psi_{R/L} = P_{R/L}\psi$. Depending on the number of fermions and their other symmetries, different symmetries can be chiral. For instance, at vanishing Yukawa couplings, the quark sector of the Standard Model features an $SU(N_\mathrm{F})_L \times SU(N_\mathrm{F})_R$ symmetry, which rotates the N_F quark flavors into each other separately for left- and right-handed components. A mass term $m_\psi \bar{\psi}\psi = 1/2 m_\psi \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right)$ breaks this symmetry explicitly; see also section "Global Symmetries Persist and Have Phenomenological Consequences". If chiral symmetry is broken - explicitly or spontaneously - at an energy scale $k_{\chi\mathrm{SB}}$, quantum fluctuations automatically generate a mass term for fermions, since it is allowed by the symmetries of the theory; see the discussion in section "Global Symmetries Persist and Have Phenomenological Consequences". Assuming that there is no fine-tuning, the generated, dimensionless mass is of order one at $k_{\chi\mathrm{SB}}$. Therefore, they decouple from the RG flow at $k_{\chi\mathrm{SB}}$ and their dimensionful masses are $\bar{m} = m \cdot k_{\chi\mathrm{SB}}$, where m is of order one. Note that within the functional RG, this decoupling happens automatically due to the built-in threshold effects. Within perturbative RG schemes, the corresponding decoupling and matching at this scale has to be done by hand.

标准模型中费米子的质量范围从几百 keV 到几 GeV，因此远轻于普朗克尺度。其原因是标准模型的手征对称性仅在电弱尺度自发破缺。手征对称性允许对左手费米子 ψ_L 和右手费米子 ψ_R 分别做旋转变换，狄拉克费米子的左手和右手分量可以通过投影算符 $P_{R/L} = \frac{1}{2}(1 \pm \gamma_5)$ 提取为 $\psi_{R/L} = P_{R/L}\psi$ 。根据费米子的数量和其他对称性，不同的对称性都可以成为手征对称性。例如，当汤川耦合为零时，标准模型的夸克部分具有 $SU(N_\mathrm{F})_L \times SU(N_\mathrm{F})_R$ 对称性，该对称性对手征分量和右手分量分别做变换，将 N_F 种夸克味互相旋转。质量项 $m_\psi\bar{\psi}\psi = 1/2m_\psi\left(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R\right)$ 会明显破缺该对称性，参见章节“整体对称性存续且具有唯象学后果”。如果手征对称性在能标 $k_{\chi\mathrm{SB}}$ 发生明显或自发破缺，量子涨落会自动生成费米子质量项，因为该质量项被理论的对称性允许，参见章节“整体对称性存续且具有唯象学后果”的讨论。在不假设精细微调的情况下，生成的无量纲质量在 $k_{\chi\mathrm{SB}}$ 处为一阶小量，因此费米子在 $k_{\chi\mathrm{SB}}$ 处从重整化群流退耦，其有量纲质量为 $\bar{m} = m \cdot k_{\chi\mathrm{SB}}$ ，其中 m 是一阶小量。注意在泛函重整化群中，这种退耦会因内置的阈效应自动发生。在微扰重整化群方案中，必须手动完成该能标下对应的退耦与匹配。

Therefore, light fermions in asymptotically safe quantum gravity can only be accommodated if gravity does not break chiral symmetry. This breaking of chiral symmetry could in principle occur on different levels (not all of which are relevant for SM fermions, because some of them would be in conflict with SU(2) gauge symmetry): first, interactions that break chiral symmetry explicitly (such as a mass term) could be induced by quantum fluctuations; second, the chirally symmetric subspace could feature no fixed point, necessitating that an asymptotically safe theory contains chiral-symmetry-breaking couplings; third, chiral symmetry might be broken spontaneously during the flow toward the IR; and fourth, the background spacetime might break chiral symmetry. In this last possibility, thermal fluctuations (as they are relevant, e.g., in the very early universe) could also play a role. We will now discuss these four possibilities.

因此，只有当引力不破缺手征对称性时，才能在渐近安全量子引力中容纳轻费米子。这种手征对称性破缺原则上可以发生在多个层面 (并非所有层面都与标准模型费米子相关，因为部分层面会与 SU(2) 规范对称性冲突): 第一，量子涨落可以诱发明显破缺手征对称性的相互作用 (例如质量项)；第二，手征对称子空间可能不存在不动点，因此渐近安全理论必须包含破缺手征对称性的耦合；第三，手征对称性可能在向红外流动的过程中自发破缺；第四，背景时空可能破缺手征对称性，在最后这种情况中，热涨落 (例如极早期宇宙中的相关热涨落) 也可能发挥作用。我们接下来讨论这四种可能性。

First, we consider the explicit breaking of chiral symmetry by induced interactions. In asymptotically safe gravity, there are no indications that this occurs. Technically, this is because in all studies so far, the coefficient β_0 in (9) vanishes for interactions that break chiral symmetry explicitly [93,96], unless a regulator is chosen that explicitly breaks chiral symmetry.

首先，我们讨论由诱发相互作用导致的手征对称性明显破缺。目前在渐近安全引力中，没有迹象表明这种情况会发生。从技术上来说，原因是迄今为止所有研究都表明，对于明显破缺手征对称性的相互作用，式 (9) 中的系数 β_0 为零 [93,96]，除非选择了会明显破缺手征对称性的正则化因子。

Second, we focus on chirally symmetric, induced interactions and ask whether they have a fixed point. In all studies to date, the answer is positive [85,91,111,130, 131]. Just like for other matter fields, gravity induces self-interactions for fermions. Fermions are special with respect to the induced self-interactions, since the induced four-fermion interactions with the lowest mass dimension are of dimension six. This is a lower dimension than for induced interactions in the other sectors. One could therefore expect that gravity can more easily make the corresponding couplings relevant. These interactions are of the form:

其次，我们聚焦于手征对称的诱导相互作用，探究它们是否存在不动点。迄今为止所有研究都给出了肯定的答案 [85,91,111,130, 131]。和其他物质场一样，引力会诱发费米子的自相互作用。就诱导自相互作用而言，费米子有其特殊性: 质量维度最低的诱导四费米子相互作用维度为六，低于其他 sector 中诱导相互作用的维度。因此可以预期，引力更容易使对应的耦合成为相关算符。这类相互作用的形式为:

$$S_{\text{Ferm, int.}} = \frac{1}{2k^2}\int \mathrm{d}^4x\sqrt{g}\,(\lambda_-(V-A)+\lambda_+(V+A)), \tag{16}$$

with

其中

$$V = \left(\bar{\psi}^i\gamma_\mu\psi^i\right)\left(\bar{\psi}^j\gamma^\mu\psi^j\right),\ A = -\left(\bar{\psi}^i\gamma_\mu\gamma_5\psi^i\right)\left(\bar{\psi}^j\gamma^\mu\gamma_5\psi^j\right). \tag{17}$$

Note that while both $\lambda_\pm$ are induced by gravitational fluctuations, there is a different basis, where only one interaction is induced [111, 130], while the second linearly independent four-fermion interaction can be set to zero consistently. Hence, this four-fermion interaction is the only example discovered so far, where gravitational fluctuations do not induce an interaction that satisfies the symmetries of the kinetic term.

请注意，虽然两个 $\lambda_\pm$ 都是引力涨落诱发的，但存在另一个基，其中仅有一种相互作用被诱发 [111, 130]，第二个线性无关的四费米子相互作用可以被一致地设为零。因此，这种四费米子相互作用是迄今为止发现的唯一一个满足动项对称性的相互作用不被引力涨落诱发的例子。

Explicit studies of the above chirally symmetric four-fermion interactions confirm that gravitational fluctuations induce those [130]; see also [85,111]. However, induced four-fermion interactions do not feature an excluded strong gravity regime, where the chirally symmetric subsector would be UV incomplete. This is independent of the strength of gravitational fluctuations. Thus, also the second possibility for chiral-symmetry-breaking is ruled out in the studies to date. This can be seen by inspecting the two beta functions $\beta_{\lambda_\pm}$, which have four fixed points for any positive value of G and any value of Λ :

对上述手征对称四费米子相互作用的明确研究证实，引力涨落确实诱发了这类相互作用 [130]；另见文献 [85,111]。但诱发的四费米子相互作用不存在被排除的强引力区域——即该手征对称子区不会在紫外不完备，这一结论与引力涨落的强度无关。因此，迄今为止的研究也排除了手征对称性破缺的第二种可能性。这一点可以通过考察两个 β 函数 $\beta_{\lambda_\pm}$ 看出，对于任意正的 G 和任意的 Λ ，这两个 β 函数存在四个不动点:

$$\beta_{\lambda_\pm} = 2\lambda_\pm + M_\pm - \frac{5\lambda_\pm G}{8\pi(1-2\Lambda)^2} \pm \frac{5G^2}{8(1-2\Lambda)^3} + \frac{3\lambda_\pm G}{4\pi(3-4\Lambda)} + \frac{15\lambda_\pm G}{8\pi(3-4\Lambda)^2}, \tag{18}$$

where the matter contributions $M_\pm$ are given by

其中物质贡献 $M_\pm$ 由下式给出

$$M_+ = \frac{8\lambda_+ (\lambda_- (N_F + 1))}{32\pi^2},\ M_- = \frac{4\lambda_-^2 (N_F - 1) + 4\lambda_+^2 N_F}{32\pi^2}. \tag{19}$$

These beta functions admit four partial fixed points, one of which is the shifted Gaussian fixed point of interest, for all values of G and Λ .

这些 β 函数共有四个部分不动点，对于所有 G 和 Λ 的取值，其中一个就是我们关心的平移高斯不动点。

Next, we consider the spontaneous breaking of chiral symmetry by gravitational fluctuations; see Fig. 11 for an illustration. The spontaneous breaking of chiral symmetry is linked to the formation of bound states, as we will explain below. Because gravity is an attractive force, one may expect that it favors bound-state formation. However, explicit calculations show that this intuition, based on the classical nature of gravity, fails to correctly predict the effect of quantum gravitational fluctuations.

接下来，我们讨论引力涨落引发的手征对称性自发破缺；示意图可见图 11。我们下文将会解释，手征对称性自发破缺与束缚态的形成相关。由于引力是吸引力，人们可能会认为引力更有利于形成束缚态。但明确计算表明，这种基于引力经典性质的直觉无法正确预测量子引力涨落的效应。

To understand the relation between the spontaneous breaking of chiral symmetry, the associated massless Goldstone boson and the induced four-fermion interactions $\lambda_\pm$, we first perform a Fierz transformation into a scalar-pseudoscalar basis. Focusing on λ_+ , the transformation reads [138, 139]

为了理解手征对称性自发破缺、对应的无质量戈德斯通玻色子与诱导四费米子相互作用 $\lambda_\pm$ 之间的关系，我们先做费尔兹变换，将其转化为标量-赝标量基。针对 λ_+ ，变换结果为 [138, 139]

$$(V + A) = -\frac{1}{2}\left[\left(\bar{\psi}^i \psi^j\right)^2 - \left(\bar{\psi}^i \gamma_5 \psi^j\right)^2\right]. \tag{20}$$

In this basis, the four-fermion interactions can be rewritten in terms of auxiliary fields using a Hubbard-Stratonovich transformation, i.e., a change of fields in the path integral; see, e.g., [139]. Focusing on the case of a single flavor for illustration, the scalar part of the four-fermion interaction can be rewritten as

在该基下，我们可以通过哈伯德-斯特拉托诺维奇变换 (即路径积分中的场变换)，利用辅助场重写四费米子相互作用；例如见文献 [139]。为便于说明，我们聚焦单味轻子的情形，四费米子相互作用的标量部分可以重写为

$$-\frac{\lambda_\psi}{4}\left(\bar{\psi}\psi\right)^2 = \left[h\left(\bar{\psi}\psi\right)\phi + m_\phi^2\phi^2\right]_{\mathrm{EoM}(\phi)}, \tag{21}$$

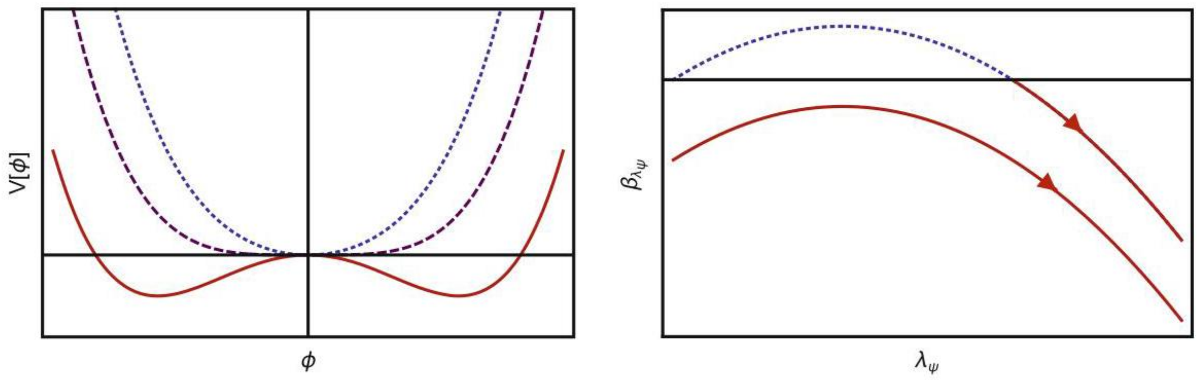


Fig. 11 We show the schematics of spontaneous chiral symmetry breaking: The effective potential for the bound-state field ϕ has a nontrivial minimum - breaking chiral symmetry, because a vev for ϕ corresponds to a vev for the fermion bilinear $\bar{\psi}\psi$ - after going through the point $m_\phi^2 = 0$ (purple dashed curve). In turn, the mass is inversely proportional to λ_ψ . Thus, the beta function in red corresponds to a chirally broken regime, because λ_ψ diverges. For the other beta function, the initial conditions can be chosen in the blue dashed region, corresponding to finite λ_ψ and thus unbroken chiral symmetry. The left fixed point corresponds to the sGFP, while the other one is interacting, even in the absence of additional fields

图 11 我们展示了手征对称性自发破缺的示意图: 经过点 $m_\phi^2 = 0$ 后 (紫色虚线)，束缚态场 ϕ 的有效势存在一个非平凡极小值——即手征对称性发生破缺，因为 ϕ 的真空期望值对应费米子双线性 $\bar{\psi}\psi$ 的真空期望值。进一步，质量与 λ_ψ 成反比。因此，红色的 β 函数对应手征破缺区域，因为 λ_ψ 发散。对于另一条 β 函数，初始条件可以选在蓝色虚线区域，对应有限的 λ_ψ，因此手征对称性未破缺。左侧不动点对应 sGFP，即使没有额外场，另一个不动点也是相互作用的

where

其中

$$m_\phi^2 = \frac{h^2}{\lambda_\psi}, \tag{22}$$

and where h is some arbitrary, real, constant, which holds on the equation of motion for the scalar field ϕ , i.e., when the auxiliary field ϕ is integrated out in the path integral.

且 h 是任意实常数，该式在标量场 ϕ 的运动方程成立，即对应路径积分中积掉辅助场 ϕ 的情况。

In terms of the scalar field ϕ , chiral symmetry is spontaneously broken, when the mass term m_ϕ^2 becomes negative. Since the mass term is related to the four-fermion interaction (see (22)), the onset of chiral symmetry breaking is indicated by a divergence of the four-fermion interaction. This argument, which we exemplified on a single channel and for a single flavor, generalizes to the full system of N_F flavors; see also [139] for a review. Hence, if the four-fermion interaction diverges, quantum gravity has broken chiral symmetry. This could happen (and does happen, e.g., in QCD [139]), when the beta function for the four-fermion interaction has no fixed points and is negative. Then, under the RG flow toward the infrared, the four-fermion interaction diverges. However, since gravitational fluctuations do not induce a fixed-point collision of the sGFP, the four-fermion interaction cannot grow beyond a bound. Hence, chiral symmetry is not broken spontaneously by gravitational fluctuations.

对于标量场 ϕ，当质量项 m_ϕ^2 变为负值时，手征对称性发生自发破缺。由于质量项与四费米子相互作用相关 (见 (22) 式)，四费米子相互作用发散标志着手征对称性破缺开始发生。我们在单通道、单味情形下给出的这个论证可以推广到 N_F 味的完整体系；综述参见文献 [139]。因此，若四费米子相互作用发散，量子引力就破缺了手征对称性。当四费米子相互作用的 β 函数没有不动点且取值为负时，这种情况就会发生 (例如在量子色动力学中确实会发生 [139])。此时，在流向红外的重整化群流作用下，四费米子相互作用会发散。但由于引力涨落不会引发规范高斯不动点 (sGFP) 发生不动点碰撞，四费米子相互作用无法增长到超出一个界的程度。因此，手征对称性不会被引力涨落自发破缺。

This is very different to fluctuations of gauge fields, which induce the same four-fermion interactions and indeed give rise to a fixed-point collision during the flow toward the IR, which can be related with spontaneous chiral symmetry breaking; see, e.g., [138-141].

这与规范场的涨落截然不同: 规范场涨落会产生相同的四费米子相互作用，并且确实会在流向红外的过程中引发不动点碰撞，这与自发手征对称性破缺相关；例如参见 [138-141]。

Combining the result from the gravitational and the gauge sector, one can obtain a lower bound on the number of fermions: if one assumes that an interacting fixed point for the Abelian gauge coupling is realized (see section ”Gauge Couplings in the Standard Model” for details), this can indeed give rise to broken chiral symmetry: gravitational fluctuations set a fixed-point value $g_{y,*}$ for the gauge coupling, which, in analogy to the situation in QCD, can induce a fixed-point collision in $\lambda_\pm$. This can be seen from the additional contributions proportional to the gauge coupling that arise in Eq. (18), where a contribution $\sim g_y^4$ arises in β_{λ_+} and a contribution $\sim -g_y^4$ in β_{λ_-} .

结合引力部分和规范部分的结果，我们可以得到费米子数的一个下界: 如果假设阿贝尔规范耦合存在相互作用不动点 (详情参见“标准模型中的规范耦合”一节)，这确实会引发手征对称性破缺: 引力涨落为规范耦合设定了一个不动点值 $g_{y,*}$，类比量子色动力学中的情形，这可以在 $\lambda_\pm$ 中引发不动点碰撞。这一点可以从 (18) 式中与规范耦合成正比的额外贡献看出: 其中 $\sim g_y^4$ 出现在 β_{λ_+} 中，$\sim -g_y^4$ 出现在 β_{λ_-} 中。

Since $g_{y,*}$ decreases when increasing the number of fermions in the system, this mechanism gives a lower bound on the number of fermions such that chiral symmetry is unbroken; see [131]. Similarly, if the presence of topology-changing gravitational instantons is assumed, chiral symmetry can also become anomalous and be broken spontaneously [142].

由于系统中费米子数增加时 $g_{y,*}$ 会减小，该机制给出了保证手征对称性不被破缺的费米子数下界；参见 [131]。类似地，如果假设存在改变拓扑的引力瞬子，手征对称性也会变成反常并发生自发破缺 [142]。

The fourth possibility through which quantum gravity might break chiral symmetry is via the background geometry. This is a mechanism which is already active for fermions on classical spacetimes: an anti-deSitter background adds an ”effective” negative mass term for fermions and thus breaks chiral symmetry through gravitational catalysis. One can think of this mechanism as follows: the effective potential for the fermion bilinear (or the corresponding scalar field) depends on the background geometry. An anti-deSitter geometry of sufficiently negative curvature favors a nontrivial ground state, and thus chiral symmetry breaking, over the trivial ground state [143, 144]. The pertinent quadratic term of the effective potential for the scalar ϕ is given by [145]

量子引力破缺手征对称性的第四种可能是通过背景几何实现。该机制在经典时空上的费米子系统中就已经生效: 反德西特背景会为费米子添加一个“有效”负质量项，从而通过引力催化作用破缺手征对称性。我们可以这样理解该机制: 费米子双线性 (或对应标量场) 的有效势依赖于背景几何。曲率足够负的反德西特几何相比平庸基态更偏好非平庸基态，因而引发手征对称性破缺 [143, 144]。标量 ϕ 有效势的相关二次项由下式给出 [145]

$$U(\phi) = -N_f \phi^2 \left(\# \frac{|R|^{3/2}}{k_{\mathrm{IR}}} + \xi \, |R| \right), \tag{23}$$

with $R < 0$ and where # is a positive number that depends on the details of the regularization. This gives rise to a curvature bound that parametrically depends on the nonminimal coupling ξ.

其中包含 $R < 0$，# 是一个正数，依赖于正规化的具体细节。由此得到一个曲率界，其参数依赖于非最小耦合 ξ。

Of course, our universe is not an anti-deSitter spacetime. Nevertheless, gravitational catalysis can become relevant, because the small-scale geometry of quantum spacetime could have both negative- and positive-curvature regions and anti-deSitter is an effective description of simple negative-curvature regions.

当然，我们的宇宙并不是反德西特时空。尽管如此，引力催化仍然可能发挥作用: 量子时空的小尺度几何可以同时存在负曲率区域和正曲率区域，而反德西特就是简单负曲率区域的有效描述。

Using this reasoning, in [145, 146], the curvature bounds were used to constrain asymptotically safe gravity: because an increasing number of fermions shifts the (background) cosmological constant to negative values, and thus shift the background curvature to large negative values, gravitational catalysis may become active at large enough fermion numbers. Thus, this mechanism gives rise to an upper bound on the number of fermions which can be light [145, 146]. The exact value of this upper bound depends on the assumed structure of the space-time geometry and on the presence and strength of thermal fluctuations [145, 146].

按照这一思路，文献 [145, 146] 利用曲率界约束渐近安全引力: 费米子数增加会将 (背景) 宇宙常数推到负值，进而将背景曲率推到大负值，因此引力催化在费米子数足够大时会被激活。因此该机制给出了可以保持轻费米子性质的费米子数上界 [145, 146]。该上界的具体数值依赖于对时空几何结构的假设，以及热涨落的存在性和强度 [145, 146]。

Gravity-Matter Systems in $d \neq 4$ Dimensions

$d \neq 4$ 维中的引力-物质系统

Synopsis: Experiments support the hypothesis that we live in a $3+1$-dimensional spacetime. We do not, however, know, why this is the case or whether it could be different. Here, we review evidence that asymptotic safety of gravity with Standard Model matter may not be achievable in dimensions much beyond four, i.e., evidence that the predictive power of asymptotic safety may extend to free parameters of the geometry of spacetime.

> 概要: 实验支持我们生活在 3+1 维时空这一假设。但我们并不清楚为何会是如此，也不知道时空维度是否本可以不同。本文我们回顾相关证据: 含标准模型物质的引力渐近安全在远超过四维的维度中可能无法实现，也就是说，有证据表明渐近安全的预言能力可以延伸到时空几何的自由参数。

Current observations indicate that our universe is four dimensional (or rather, 3+1 dimensional), at least down to length scales corresponding to an energy of $\sim$ 10TeV . Accordingly, our universe might be of higher dimension in the deep UV, if the additional dimensions are compact and therefore inaccessible at low energies. Note that here, we refer to the topological dimension. There are in fact indications that other notions of dimensionality, most importantly the spectral dimension, instead exhibit a dynamical reduction in the UV [57, 147-149]. Such different notions of dimensionality are not related to each other and may thus exhibit differences in the UV. Indeed, in string theory, such extra topological dimensions are necessary for the internal consistency of the theory. It is therefore interesting to understand what the status of extra dimensions is in other approaches to quantum gravity.

> 当前观测表明我们的宇宙是四维的 (更准确地说是 3+1 维)，至少在对应于能量 $\sim$ 10TeV 的长度尺度下是如此。因此，如果额外维度是紧致的、因而在低能下不可观测，那么我们的宇宙在深紫外区可能是更高维的。请注意，本文此处讨论的是拓扑维度。实际上已有迹象表明，其他维度定义 (最重要的是谱维度) 反而在紫外区会发生动力学降维 [57, 147-149]。这些不同的维度定义彼此无关，因此在紫外区可能表现出差异。事实上，弦理论中这类额外拓扑维度是理论自洽性的必要条件。因此，探究额外维度在其他量子引力研究方案中的状态是很有意义的。

The compatibility of extra dimensions with the asymptotic-safety paradigm for quantum gravity and matter has been tested by studying (i) the impact of matter fields on the gravitational fixed point (ii) mechanisms for a UV complete matter sector, and (iii) gravitational contributions to LHC scattering amplitudes to connect to observational constraints on extra dimensions. We will briefly discuss these results in the following.

> 额外维度与量子引力和物质的渐近安全范式的兼容性已经通过以下三方面研究得到检验:(i) 物质场对引力不动点的影响；(ii) 紫外完备物质部门的机制；(iii) 引力对 LHC 散射振幅的贡献，以建立与额外维度观测约束的联系。下文我们将简要讨论这些研究结果。

First, the coefficients b_{grav} and a_i in (5) are dimension dependent. All studies so far indicate that gravitational contributions remain antiscreening in $d > 4$, i.e., $b_{\text{grav}} > 0$ in (5) [50, 150, 151] , such that pure gravity can become asymptotically safe in a larger number of dimensions. Since the gravitational contribution and the matter contributions scale differently with the dimensionality (see [80] for an explicit example), bounds on the number of matter fields may arise in $d > 4$. It may thus be the case that asymptotic safety of gravity with Standard Model matter is not achievable in $d > 4$; see [80] for a first study of this question. A systematic investigation of this question, which lifts the approximations made in [80], has not been completed yet.

首先，式 (5) 中的系数 b_{grav} 和 a_i 依赖于维度。迄今为止所有研究都表明，引力贡献在 $d > 4$ 中仍然是反屏蔽效应，即式 (5) 中的 $b_{\text{grav}} > 0$ [50, 150, 151]，因此纯引力在更多维度中也可以成为渐近安全的。由于引力贡献和物质贡献随维度的标度行为不同 (具体例子参见 [80])，对物质场的数量可能会在 $d > 4$ 中产生约束。因此，含标准模型物质的引力渐近安全可能无法在 $d > 4$ 中实现；关于该问题的第一项研究参见 [80]。目前尚未完成对该问题的系统研究，来解除 [80] 中所做的近似。

Second, if we assume that the higher-dimensional theory contains a fundamental Abelian gauge coupling, demanding a UV completion constrains the number of dimensions. This is because the triviality problem in the Abelian gauge sector becomes more severe in larger dimensions, where the Abelian gauge coupling has a negative canonical mass dimension. This acts akin a screening contribution to the scale dependence of the gauge coupling. Hence, to induce asymptotic freedom in $d > 4$ in the gauge sector, the antiscreening gravitational contribution has to overcome this screening dimensional contribution; see also section "Gauge Couplings in the Standard Model". This requires that f_g (cf. Eq. (24)) increases with increasing dimensionality. Explicit studies indicate that this is only possible if gravitational fluctuations become stronger when we increase the dimensionality [132, 134, 152]. Hence, a UV completion of the Abelian gauge sector is only possible for a more strongly coupled gravity theory. However, such a strongly coupled regime might be excluded due to the WGB in the Abelian gauge sector; see section "Step 2: The Symmetry-Preserving Interactions Which Are Generated by Gravity Have an Asymptotically Safe Fixed Point". Indeed, according to the studies in [132, 134, 152] , it is not possible to reconcile the strong coupling required to solve the triviality problem with the weak coupling required to satisfy the weak-gravity bound, if $d \geq 6$.

其次，如果我们假设高维理论包含一个基本阿贝尔规范耦合，那么对紫外完备性的要求会对维度数量施加约束。这是因为阿贝尔规范部门的平庸性问题在更高维度中更为严重，在高维下阿贝尔规范耦合的正则质量维数为负。这就类似于对规范耦合的标度演化产生了一个屏蔽贡献。因此，要让规范部门的 $d > 4$ 产生渐近自由，引力的反屏蔽贡献必须克服这种由维度带来的屏蔽贡献；另见“标准模型中的规范耦合”一节。这要求 f_g (参见式 (24)) 随维度增加而增大。已有研究表明，这只有当我们增加维度时引力涨落变强才可能实现 [132, 134, 152]。因此，阿贝尔规范部门的紫外完备只有在引力耦合更强的理论中才可能实现。但这种强耦合区可能因阿贝尔规范部门的弱引力边界 (WGB) 而被排除；参见“步骤 2: 引力生成的对称保持相互作用具有渐近安全不动点”一节。实际上，根据 [132, 134, 152] 中的研究，如果 $d \geq 6$，解决平庸性问题所需的强耦合，与满足弱引力边界所需的弱耦合是无法调和的。

Third, a more phenomenological approach to extra dimensions was taken, prior to the start of the LHC, with the hope of constraining asymptotic safety in large extra dimensions by experiment. If the extra dimensions are large enough, the fundamental Planck scale is close to TeV scales; see [153]. Accordingly, scattering processes at the TeV scale would be sensitive to the production or exchange of virtual Kaluza-Klein gravitons. These would leave an imprint on scattering amplitudes of SM particles, for example, by missing energy signatures. In [154- 156], this scenario was investigated within asymptotically safe quantum gravity. Specifically, it was found that the gravitational di-lepton production at LHC energies would be well above SM backgrounds, if the fundamental Planck scale was at the TeV scale. Similarly, [157] found that gravity-mediated photon-photon scattering can rise above the SM background (which in this case is a pure loop effect, with no tree-level contribution). To date, no such signatures were discovered, constraining the radii of large extra dimensions. We caution here that the scattering cross sections in [154-156] were computed within an approximation where the RG scale k is identified with the physical momentum scale of a scattering process. A more careful investigation of gravity-mediated scattering amplitudes requires to encode the full momentum dependence of the vertices, e.g., by means of form factors; see [41, 158]. Recent studies of gravity-mediated scattering processes indicate that scattering amplitudes can indeed be finite, despite the presence of trans-Planckian modes [158]. We refer the reader to the chapter on form factors and scattering amplitudes [42] for details.

第三，在 LHC 启动前，学界对额外维度采取了更偏唯象的研究方式，希望通过实验约束大额外维度下的渐近安全。若额外维度足够大，基本普朗克尺度会接近 TeV 标度，参见文献 [153]。相应地，TeV 能标的散射过程会对虚卡鲁扎-克莱因引力子的产生或交换敏感。这类引力子会在标准模型粒子的散射振幅中留下印记，例如表现为丢失能量特征。文献 [154-156] 在渐近安全量子引力框架下研究了该场景，具体发现: 若基本普朗克尺度处于 TeV 能标，LHC 能区引力诱导的双轻子产生会远高于标准模型本底。类似地，文献 [157] 发现引力介导的光子-光子散射可以超出标准模型本底 (该本底是纯圈效应，不存在树图贡献)。截至目前，人们尚未发现这类特征，因此约束了大额外维度的半径。我们在此提醒: 文献 [154-156] 中的散射截面是在将 RG 标度 k 等同于散射过程物理动量标度的近似下计算得到的。要更严谨地研究引力介导的散射振幅，需要对顶角的完整动量依赖进行编码，例如通过形状因子实现，参见文献 [41, 158]。近期对引力介导散射过程的研究表明，即便存在跨普朗克模式，散射振幅也确实可以是有限的 [158]，细节请读者参考形状因子与散射振幅相关章节 [42]。

At a more formal level, it is also of interest to explore the gravitational dressing of matter theories with asymptotic safety in $d = 3$ or $d = 2$. Such matter theories encode universal critical behavior at continuous phase transitions; with the Wilson-Fisher fixed point a paradigmatic example. For such systems in statistical physics, gravitational fluctuations are not expected to be relevant, because the Planck length is much smaller than the atomic scale in those systems, where the quantum-field-theoretic description breaks down. Nevertheless, it is of interest to understand whether such universality classes persist and are gravitationally dressed. This sheds light on how two distinct, asymptotically safe theories can be brought together. It may even, within an AdS/CFT-type of correspondence, become of interest for quantum gravity in a more indirect way.

在更形式化的层面，探索渐近安全框架下物质理论的引力修饰也很有意义，相关场景包括 $d = 3$ 或 $d = 2$ 。这类物质理论可以描述连续相变处的普适临界行为，威尔逊-费希尔不动点就是典型例子。对于统计物理中的这类系统，引力涨落一般被认为不相关: 因为在这类系统中，普朗克长度远小于原子尺度，而量子场论描述在原子尺度就已经失效了。但探究这类普适类是否能在引力修饰下仍然存在仍有意义，这能帮助我们理解两个不同的渐近安全理论如何结合。甚至在 AdS/CFT 类型的对应中，它还可能以更间接的方式对量子引力研究产生价值。

Explicit studies of gravitationally dressed universality classes have started from the Wilson-Fisher fixed point [81] and considered universality classes in three-dimensional $O(N)$ models in [83], finding evidence that these universality classes can be dressed gravitationally.

针对引力修饰普适类的 explicit 研究从威尔逊-费希尔不动点出发展开 [81]，文献 [83] 进一步研究了三维 $O(N)$ 模型中的普适类，找到了这些普适类可以保留引力修饰的证据。

Toward a UV Completion of the Standard Model

对标准模型紫外完备性的探索

Synopsis: The Standard Model is not UV complete on its own. Under the impact of asymptotically safe gravity, it may become UV complete. The current state of the art sees two distinct possibilities for this UV completion: either, all SM couplings vanish in the far UV; or some of them are nonzero. Except for the Higgs self-coupling, all couplings that vanish in the far UV correspond to free parameters of the theory. Therefore, an asymptotically safe gravity-matter theory has at least one free parameter less in the SM sector than the SM on its own. Each coupling that is nonzero in the far UV changes its status from a free parameter to a calculable quantity. Thereby, quantities like the low-energy value of the Abelian gauge coupling or the top Yukawa coupling may be calculable from first principles. In turn, these calculations provide observational tests of the asymptotically safe theory.

提要: 标准模型本身并非紫外完备的。在渐近安全引力的作用下，它可以成为紫外完备理论。目前该领域的研究认为，这种紫外完备存在两种不同的可能性: 要么所有标准模型耦合在极紫外区都为零；要么部分耦合不为零。除希格斯自耦合外，所有在极紫外区为零的耦合都对应理论的自由参数。因此，渐近安全引力-物质理论的标准模型部分，至少比独立的标准模型少一个自由参数。每一个在极紫外区非零的耦合都会改变自身性质，从自由参数变为可计算量。据此，阿贝尔规范耦合或顶夸克汤川耦合的低能值这类物理量都可以从第一性原理计算得到。反过来，这些计算也可以为渐近安全理论提供观测检验。

In the future, when the systematic uncertainty of calculations is decreased further, only one or none of these two possibilities will remain phenomenologically viable, because the fixed points at nonzero values generate upper bounds on the values of the couplings. If these upper bounds are lower than the measured values, then neither the fixed point at zero value nor the fixed point at nonzero value remains accessible. There is thus the distinct possibility of ruling out an asymptotically safe theory of gravity and the SM. Given the lack of experimental guidance on quantum gravity, such a result would be important progress.

未来，随着计算系统误差进一步降低，两种可能性中将仅有一种甚至零种符合唯象要求，因为非零不动点会对耦合取值给出上限。如果该上限低于实验测量值，那么零不动点和非零不动点都不再成立。因此，完全有可能排除引力与标准模型的渐近安全理论。在量子引力缺乏实验指引的现状下，这样的结果将是重要的进展。

The SM accurately describes all visible matter at low energies and has been experimentally tested up to TeV scales. Nevertheless, the SM is only an effective description of matter, which breaks down at high energies. Note that this is due to Landau poles in the Higgs sector [159] and Abelian gauge sector [160]. It remains a theoretical possibility that there is a nonperturbative UV completion of the SM as a whole, even though these separate sectors are not UV complete due to a triviality problem. There is, however, no indication that this theoretical possibility is realized. Due to the breakdown of the SM at high energies, it is therefore crucial to investigate under which conditions asymptotically safe quantum gravity can UV complete the SM. On the one hand, this is an important consistency test for any theory that aims to describe nature on the fundamental level. On the other hand, such an asymptotic-safety UV completion may be more predictive than the SM, allowing to calculate some of the SM's free parameters from first principles. This might even provide answers to some long-standing questions in particle physics, such as the origin of different masses for fermions.

标准模型准确描述了低能区所有可见物质，且在 TeV 能标范围内都得到了实验验证。尽管如此，标准模型只是对物质的有效描述， 在高能区会失效: 这一失效源于希格斯场域 [159] 和阿贝尔规范场域 [160] 中的朗道极点。尽管这些独立场域因平庸性问题不具备紫外完备性，整体而言标准模型存在非微扰紫外完备性仍然是理论上的可能，只不过目前没有迹象表明这一理论可能性会成真。由于标准模型在高能区失效, 因此探究渐近安全量子引力能在哪些条件下为标准模型提供紫外完备性至关重要。一方面， 对于任何旨在从基本层面描述自然的理论而言， 这都是一项重要的自洽性检验; 另一方面，这种渐近安全紫外完备性可能比标准模型具备更强的预测能力，允许我们从第一性原理出发计算部分标准模型的自由参数。这甚至有望解答粒子物理学中一些悬而未决的长期问题，例如费米子质量差异的起源。

In the following, we will first briefly summarize the Landau pole/triviality problems in sectors of the SM and review how asymptotically safe quantum gravity might cure these.

下文我们将首先简要总结标准模型各 sector 的朗道极点/平庸性问题， 再回顾渐近安全量子引力如何解决这些问题。

The Landau poles/triviality problems arise in some of the canonically marginal couplings of the SM which we write as c here (later, the gauge couplings will be $g_{Y/2/3}$, the Yukawa couplings y_f , and the Higgs self-interaction will be λ). Their scale dependence takes the following schematic form:

朗道极点/平庸性问题出现在标准模型的部分正则边缘耦合中， 我们在此将其记为 c (后续规范耦合记为 $g_{Y/2/3}$ ， 汤川耦合记为 y_f ， 希格斯自相互作用记为 λ)。这些耦合的标度依赖满足以下示意形式:

$$\beta_c = -f_c c + \beta_{c,1} c^n + \mathcal{O}\left(c^{n+1}\right), \tag{24}$$

where $n = 2$ for the quartic Higgs self-interaction and $n = 3$ for the gauge and Yukawa couplings. Here, $\beta_{c,1}$ is the one-loop matter contribution, and f_c is the gravitational contribution to the scale dependence of the respective coupling. The signs of the gravitational and the matter contributions differ from sector to sector, giving rise to distinct phenomenological implications for each of them. We will see that three of the four possibilities in Table 3 may be realized in the SM with gravity.

其中 $n = 2$ 对应四次希格斯自相互作用， $n = 3$ 对应规范耦合与汤川耦合。$\beta_{c,1}$ 是单圈物质贡献， f_c 是引力对对应耦合标度依赖的贡献。引力贡献和物质贡献的符号随 sector 不同而变化，因此各情况有不同的唯象推论。我们将看到， 表 3 中的四种可能性里有三种可以在含引力的标准模型中实现。

In the absence of gravity, $\beta_{c,1} > 0$ implies that the coupling shrinks under the RG flow to the IR. Starting with a finite value in the very far UV, $k \to \infty$, the IR value of the coupling therefore vanishes, i.e., the theory becomes noninteracting or trivial. This is called the triviality problem. In perturbation theory, the triviality problem is related to a Landau pole in the UV: when one attempts to shift the scale of the Landau pole to infinity, the IR value of the coupling is required to be zero, i.e., the theory becomes trivial. Beyond perturbation theory, nonperturbative studies have confirmed that there is a triviality problem in the quartic scalar and the Abelian gauge coupling; see section ”Gauge Couplings in the Standard Model” for a more detailed discussion.

在不存在引力的情况下，$\beta_{c,1} > 0$ 意味着耦合在重整化群流流向红外时不断减小。如果在极紫外区初始耦合取有限值 $k \to \infty$，那么耦合的红外值就会变为零，也就是理论变得无相互作用，即平庸，这就是平庸性问题。在微扰论中，平庸性问题和紫外区的朗道极点相关: 当人们试图将朗道极点的能标移到无穷远时，就要求耦合的红外值为零，也就是理论变得平庸。在微扰论之外，非微扰研究已经证实四次标量耦合和阿贝尔规范耦合存在平庸性问题，更详细的讨论可以参见“标准模型中的规范耦合”一节。

Table 3 We show the four distinct possibilities for the signs of coefficients in Eq. (24): In the first line, matter and gravity are both antiscreening, such that there is only an asymptotically free fixed point. This situation may be realized for non-Abelian gauge couplings; see section ”Gauge Couplings in the Standard Model”. In the second line, matter is antiscreening and gravity screening, such that there are two fixed points, with the one at vanishing coupling generating a prediction. In the third line, matter is screening and gravity is antiscreening, such that there are two fixed points, with the one at nonvanishing coupling generating a prediction. This situation may be realized for the Abelian gauge coupling and for Yukawa couplings; see sections ”Gauge Couplings in the Standard Model” and ”Yukawa Couplings in the Standard Model”. In the fourth line, matter and gravity are both screening, such that there is only the free fixed point which generates a prediction. In this case, the matter theory may be trivial, unless additional interactions can regenerate the coupling, even though it is required to vanish at the Planck scale in order for the theory to be UV complete. This may be realized for the Higgs self-interaction; see section ”Higgs Quartic Coupling in the Standard Model”

表 3 我们展示了式 (24) 中系数符号的四种不同情形: 第一行中，物质与引力均为反屏蔽，因此仅存在一个渐近自由不动点。这种情况可出现在非阿贝尔规范耦合中；参见章节“标准模型中的规范耦合”。第二行中，物质为反屏蔽、引力为屏蔽，因此存在两个不动点，其中耦合为零的不动点给出一个预言。第三行中，物质为屏蔽、引力为反屏蔽，因此存在两个不动点，其中非零耦合的不动点给出一个预言。这种情况可出现在阿贝尔规范耦合与汤川耦合中；参见章节“标准模型中的规范耦合”和“标准模型中的汤川耦合”。第四行中，物质与引力均为屏蔽，因此仅存在自由不动点，由该不动点给出一个预言。这种情况下，物质理论可能是平庸的，除非额外相互作用可以重新生成耦合——即便为了让理论在紫外完备，该耦合必须在普朗克尺度上为零。这种情况可出现在希格斯自相互作用中；参见章节“标准模型中的希格斯四次耦合”

Sign of $\beta_{c,1}$	Sign of f_c	UV complete	c predicted at low energies from $c_* = 0$ fixed point	$c_* \neq 0$ possible	c predicted at low energies from $c_* \neq 0$ fixed point
Negative	Positive	Yes	No	No	-
Negative	Negative	Yes	Yes	Yes	No
Positive	Positive	Yes	No	Yes	Yes
Positive	Negative	Yes	Yes	No	-

The physical interpretation of the Landau pole/triviality problem is that the SM is an effective field theory with a finite range of validity. It breaks down if one attempts to extrapolate it toward the UV past the scale of the Landau poles. Because those Landau poles are transplanckian in the SM, this problem is often ignored in particle physics. Note that the Landau poles are transplanckian for the measured value of the Higgs mass and Abelian gauge coupling. If those values were different, the Landau poles could occur already below the Planck scale. The absence of Landau poles below the Planck scale is also an important guiding principle for physics beyond the SM. In the context of quantum gravity, one may no longer ignore the Landau pole problem and must find a solution. Such a solution may lie in the coupling to gravity - one can indeed interpret the transplanckian scale of the Landau pole as an indication that the missing new physics is nothing but gravity.

朗道极点/平庸性问题的物理解释是: 标准模型是一个仅具有有限适用范围的有效场论。如果尝试将其向紫外外推到超过朗道极点的能标，它就会失效。由于在标准模型中这些朗道极点都位于普朗克能标以上，粒子物理学中常常忽略这个问题。请注意，对于测量得到的希格斯质量和阿贝尔规范耦合数值，朗道极点确实都在普朗克能标以上。如果这些数值不同，朗道极点本就可以出现在普朗克能标以下。普朗克能标以下不存在朗道极点，也是超出标准模型新物理的重要指导原则。在量子引力的框架下，人们不能再忽略朗道极点问题，必须找到解决方案。这个解决方案可能存在于与引力的耦合中——人们确实可以将朗道极点处于普朗克能标以上这一特征，解读为缺失的新物理不是别的，正是引力。

Gauge Couplings in the Standard Model

标准模型中的规范耦合

Synopsis: The gauge sector of the SM has a problem and three riddles. The problem is the Landau pole (or triviality problem) in the Abelian gauge coupling. The riddle is, what sets the values of the three gauge couplings at low energies, in particular, what sets the value of the fine-structure constant $\alpha = 1/137$?

概要: 标准模型规范区存在一个问题和三个谜题。问题是阿贝尔规范耦合中的朗道极点 (或平庸性问题)。谜题是，是什么决定了三个规范耦合在低能下的取值，尤其是决定了精细结构常数 $\alpha = 1/137$ 的取值?

There are compelling indications that asymptotically safe gravity could solve the problem. Whether or not it may also solve one of the three riddles and explain why $\alpha = 1/137$ is less sure, because it depends on additional properties of asymptotically safe gravity.

现有充分证据表明渐近安全引力可以解决这个问题。它能否同时解开三个谜题之一并解释为什么 $\alpha = 1/137$ 目前尚无定论，因为这取决于渐近安全引力的额外性质。

The gauge sector of the Standard Model is divided into an Abelian and a non-Abelian sector. The Abelian hypercharge group $U_Y(1)$ with coupling $g_y(k)$ is, together with the non-Abelian $SU(2)$, spontaneously broken to the $U_{em}(1)$ electromagnetic gauge group with the electromagnetic coupling $e(k)$, related to the fine-structure constant $\alpha(k) = e(k)^2/(4\pi)$. This sector features a problem and a riddle: the problem is the Landau pole problem; the riddle is why the fine-structure constant takes the value $\alpha(k \to 0) = 1/137$. It is a compelling scenario that asymptotically safe gravity could solve the problem and the riddle at the same time. We review the evidence for this scenario below.

标准模型规范区分为阿贝尔和非阿贝尔两个部分。阿贝尔超荷群 $U_Y(1)$ 及其耦合 $g_y(k)$ 与非阿贝尔 $SU(2)$ 一起，自发破缺为 $U_{em}(1)$ 电磁规范群，其电磁耦合为 $e(k)$ ，与精细结构常数 $\alpha(k) = e(k)^2/(4\pi)$ 相关。该部分存在一个问题和一个谜题: 问题是朗道极点问题；谜题是为什么精细结构常数取 $\alpha(k \to 0) = 1/137$ 这个值。有观点认为渐近安全引力可以同时解决这个问题和这个谜题，这一假说很有说服力。我们在下文中回顾支持该假说的证据。

The Landau pole or triviality problem is present in the absence of gravity, i.e., with $f_{g_y} = 0$ in Eq. (24). Quantum fluctuations of charged matter, i.e., the charged lepton fields and the quarks, turn the vacuum into a screening medium, such that for the Abelian hypercharge $\beta_{g_y,1} > 0$ in (24) [161] (with $\beta_{g_y,1} = 41/\left(6 \cdot 16\pi^2\right)$). Thus, the coupling decreases when flowing toward lower energies. As a consequence, a divergent value of $g_y\left(k = \Lambda_{\text{Landau}}\right)$ is mapped to the measured value of α at low energies. This can already be seen at one-loop order. To this order, the solution to β_{g_y} reads

朗道极点或平庸性问题在不存在引力的情况下出现，即对应式 (24) 中的 $f_{g_y} = 0$ 。带电物质即带电轻子场与夸克的量子涨落将真空变成了屏蔽介质，因此对于 (24) 中 [161] 的阿贝尔超荷 $\beta_{g_y,1} > 0$ (其中存在 $\beta_{g_y,1} = 41/\left(6 \cdot 16\pi^2\right)$) 会发生这一情况。因此耦合在向低能区流动时会减小，其结果是，有限发散的 $g_y\left(k = \Lambda_{\text{Landau}}\right)$ 映射到低能区测得的 α 取值。这一点在单圈阶就可以观察到。在该阶下，β_{g_y} 的解为

$$g_y^2(k) = \frac{g_y^2(k_0)}{1 - 2\beta_{g_y,1} g_y^2(k_0) \ln\left(\frac{k}{k_0}\right)}, \tag{25}$$

where k_0 is a reference scale. Because $\beta_{g_y,1} > 0, g_y^2(k)$ diverges at a finite scale Λ_{Landau}

其中 k_0 为参考标度。因为 $\beta_{g_y,1} > 0, g_y^2(k)$ 在有限标度 Λ_{Landau} 处发散

$$\Lambda_{\text{Landau}} = \exp\left(\frac{1}{2\beta_{g_y,1} g_y^2(k_0)}\right) k_0. \tag{26}$$

In theory, this divergence, the so-called Landau pole, can be avoided by setting $g_y^2(k_0) = 0$. However, then $g_y^2(k)$ would be zero at all scales k , which is clearly in contradiction with the experimental observation of an interacting electromagnetic sector at low energies in our universe. Hence, the presence of a Landau pole in the Abelian gauge sector signals the breakdown of the Standard Model. From Eq. (26), with $e\left(k_0 = 511\text{keV}\right)$ and $\beta_{e,1} = 1/\left(12\pi^2\right)$ (i.e., looking at QED), one can infer that $\Lambda_{\text{Landau}} \approx 10^{286}$eV (which is shifted to $\sim 10^{34}$GeV for the matter content of the SM and in a two-loop calculation), which is highly transplanckian. The computation giving rise to the Landau pole was performed within a perturbative one-loop approximation. Clearly, this approximation is not expected to be accurate when g_y becomes large. However, nonperturbative studies using lattice [160,162] and functional [163] methods find indications that the triviality problem in the Abelian gauge coupling persists beyond perturbation theory.

理论上，这种发散即所谓朗道极点，可以通过设置 $g_y^2(k_0) = 0$ 避免。但此时 $g_y^2(k)$ 会在所有尺度 k 上都为零，这显然与我们宇宙低能区存在相互作用电磁 sector 的实验观测矛盾。因此，阿贝尔规范 sector 存在朗道极点标志着标准模型的破缺。根据式 (26)，代入 $e\left(k_0 = 511\text{keV}\right)$ 和 $\beta_{e,1} = 1/\left(12\pi^2\right)$ (即研究量子电动力学的情况)，可以推导出 $\Lambda_{\text{Landau}} \approx 10^{286}$eV (对标准模型的物质内容、双圈计算而言该值偏移至 $\sim 10^{34}$GeV)，其能量远在普朗克能标之上。朗道极点的计算是在微扰单圈近似下完成的。显然，当 g_y 变大时，该近似无法保证精确。不过，基于格点 [160,162] 和泛函 [163] 方法的非微扰研究显示，阿贝尔规范耦合的平凡性问题在微扰论之外依然存在。

The triviality problem indicates that new physics must exist at very high energies. As the energy scale of the Landau pole is beyond the Planck scale, quantum gravity is one candidate for such new physics. In fact, it is a compelling candidate, because it is not really "new physics," given that we know for sure that the gravitational field exists.

平凡性问题表明，极高能区必然存在新物理。由于朗道极点的能标高于普朗克能标，量子引力是这类新物理的一个候选。实际上，它是极具说服力的候选——因为引力场的存在是确凿无疑的，它并非真正意义上的“新物理”。

Asymptotically safe gravitational fluctuations contribute through f_{g_y} to the scale dependence of g_y , cf. (24). f_{g_y} depends on the gravitational couplings; explicit forms have been calculated in [87, 107, 133, 164-169].

渐近安全引力涨落通过 f_{g_y} 贡献于 g_y 的标度依赖，参见式 (24)。f_{g_y} 依赖于引力耦合，具体形式已在文献 [87, 107, 133, 164-169] 中算出。

It is crucial that the gravitational contribution is linear in g_y itself. This may be understood in two ways: first, it is a consequence of counting powers of g_y in the loop diagrams that generate the gravity contribution to β_{g_y} . Second, no lower-order contribution $\sim g_y^0$ exists, because it would break the global chiral symmetry of charged fermions, where left- and right-handed components transform under independent phase rotations. In section ”Global Symmetries Persist and Have Phenomenological Consequences”, we explain why such global symmetries prevent the generation of such low-order terms in beta functions. Therefore, for small values of the coupling, i.e., close to the Gaussian fixed point, the gravitational contribution dominates over the pure-matter contribution.

引力贡献对 g_y 本身是线性的，这一点至关重要。这可以从两方面理解: 首先，它是对圈图中 g_y 的幂次计数得到的自然结果，圈图产生引力对 β_{g_y} 的贡献。其次，不存在更低阶的贡献项 $\sim g_y^0$，因为那会破坏带电费米子的整体手征对称性——带电费米子的左手和右手分量在各自独立的相位转动下变换。在“整体对称性得以保留并具有唯象学后果”一节中，我们解释了为何这类整体对称性会阻止 β 函数生成此类低阶项。因此，当耦合取值很小 (即靠近高斯不动点) 时，引力贡献主导纯物质贡献。

Explicit computations using the FRG indicate that $f_{g_y} \geq 0$ [87, 107, 133, 164– 167, 169], such that gravitational fluctuations have an antiscreening effect on the gauge coupling. One can also argue that the gravitational contribution should have a negative sign, i.e., be antiscreening: gravity generates a self-coupling of the Abelian gauge field, such that effectively, the Abelian gauge field behaves like a non-Abelian one. This argument does not refer to the symmetry group of the local symmetry; instead, it just refers to a self-interacting vs, a non-self-interacting gauge theory. In a non-Abelian gauge theory, gauge-field fluctuations antiscreen the vacuum. Thus, one may argue, the combined effect of gravity and Abelian gauge field should also be antiscreening, and thus, $f_{g_y} > 0$ is the expected result.

利用泛函重整化群 (FRG) 的显式计算表明 $f_{g_y} \geq 0$ [87, 107, 133, 164– [167, 169]，即引力涨落对规范耦合产生反屏蔽效应。也可以论证引力贡献应为负号，即反屏蔽: 引力会生成阿贝尔规范场的自耦合，使得阿贝尔规范场的行为等效于非阿贝尔规范场。这一论证不涉及定域对称性的对称群，仅区分自相互作用与非自相互作用规范理论。在非阿贝尔规范理论中，规范场涨落对真空产生反屏蔽。因此可以推断，引力与阿贝尔规范场的联合效应也应当是反屏蔽，因此 $f_{g_y} > 0$ 是预期的结果。

The gravitational contribution f_{g_y} is a function of the dimensionless Newton coupling G_N and, in the absence of other gravitational couplings, given by

引力贡献 f_{g_y} 是无量纲牛顿耦合 G_N 的函数，且在不存在其他引力耦合时，其形式为

$$f_{g_y} = \frac{5G_N}{18\pi}. \tag{27}$$

Above the Planck scale, G_N assumes its fixed-point value, such that f_{g_y} = const. Below the Planck scale, G_N scales like k^2 , i.e., decreases toward the IR. Thus, gravitational fluctuations decouple very quickly below the Planck scale and are completely negligible at experimentally accessible scales - just as one expects. Therefore, f_{g_y} can be approximated as zero below the Planck scale, such that the scale dependence of the gauge coupling is only driven by SM fields.

普朗克能标以上，G_N 取其不动点值，满足 f_{g_y} = 为常数。普朗克能标以下，G_N 按 k^2 的形式标度变化，即向红外区递减。因此引力涨落在普朗克能标以下迅速退耦合，在实验可及能标下完全可以忽略——这完全符合预期。因此，普朗克能标以下 f_{g_y} 可近似为零，规范耦合的能标依赖仅由标准模型场驱动。

In addition to G_N, f_{g_y} depends on the other gravitational couplings, including the cosmological constant Λ and higher-order couplings [168], such as the $R_{\mu\upsilon}R^{\mu\upsilon}$ - coupling b . In terms of G_N, Λ and b, f_{g_y} reads

除 G_N, f_{g_y} 外，它还依赖其他引力耦合，包括宇宙学常数 Λ 和高阶耦合 [168]，例如 $R_{\mu\upsilon}R^{\mu\upsilon}$ 耦合 b 。用 G_N, Λ 和 b, f_{g_y} 表示即为

$$f_{g_y} = \frac{G_N}{36\pi}\frac{10 + 7b - 40\Lambda}{(1 + b - 2\Lambda)^2} \tag{28}$$

In the gravitational fixed-point regime, different scenarios are realized, depending on the sign of f_{g_y} . If $f_{g_y} < 0$, then gravitational fluctuations are screening and the Landau pole problem persists. If $f_{g_y} > 0$, then gravitational fluctuations are antiscreening. This is the scenario that appears to be realized, when using fixed-point values obtained in the literature. Thus, they compete with the screening fluctuations of charged matter fields. At small g_y , the gravitational contribution dominates and the gauge coupling becomes asymptotically free. At large g_y , the matter contribution dominates and the Landau pole problem persists. In between, at $g_y = g_{y,*}$, with

在引力不动点区域，根据 f_{g_y} 的符号不同会实现不同的情形。若 $f_{g_y} < 0$ ，则引力涨落为屏蔽效应，朗道极点问题仍然存在。若 $f_{g_y} > 0$ ，则引力涨落为反屏蔽效应。使用文献中得到的不动点值计算时，得出的就是这种情形。因此引力涨落与带电物质场的屏蔽涨落相互竞争。在小 g_y 下，引力贡献占主导，规范耦合变为渐近自由。在大 g_y 下，物质贡献占主导，朗道极点问题仍然存在。在两者之间的 $g_y = g_{y,*}$ 处，满足

$$g_{y,*} = 4\pi\sqrt{\frac{6f_{g_y}}{41}} \tag{29}$$

the screening and antiscreening effects cancel out exactly and generate an interacting fixed point.

屏蔽效应与反屏蔽效应恰好抵消，产生一个相互作用不动点。

This fixed point has predictive power: because gravity fluctuations antiscreen the coupling at $g_y < g_{y,*}$, they drive the coupling toward the fixed-point value from below; see also Fig. 12. Conversely, because matter fluctuations screen the coupling at $g_y > g_{y,*}$, they drive the coupling toward the fixed-point value from above.

Thus, the coupling stays fixed at $g_y = g_{y,*}$ all the way down to the Planck scale. Asymptotic safety thereby produces a unique value of the coupling at the Planck scale. Below the Planck scale, the SM RG flow maps this unique value to a unique value in the IR.

该不动点具有预言能力: 因为引力涨落在 $g_y < g_{y,*}$ 处对耦合反屏蔽，会从下方推动耦合趋近不动点值；参见图 12。反之，因为物质涨落在 $g_y > g_{y,*}$ 处对耦合屏蔽，会从上方推动耦合趋近不动点值。因此耦合会一直保持在 $g_y = g_{y,*}$，一直延伸到普朗克能标。渐近安全性因此在普朗克能标给出唯一的耦合值。普朗克能标以下，标准模型重整化群流会将这个唯一值映射到红外区的唯一值。

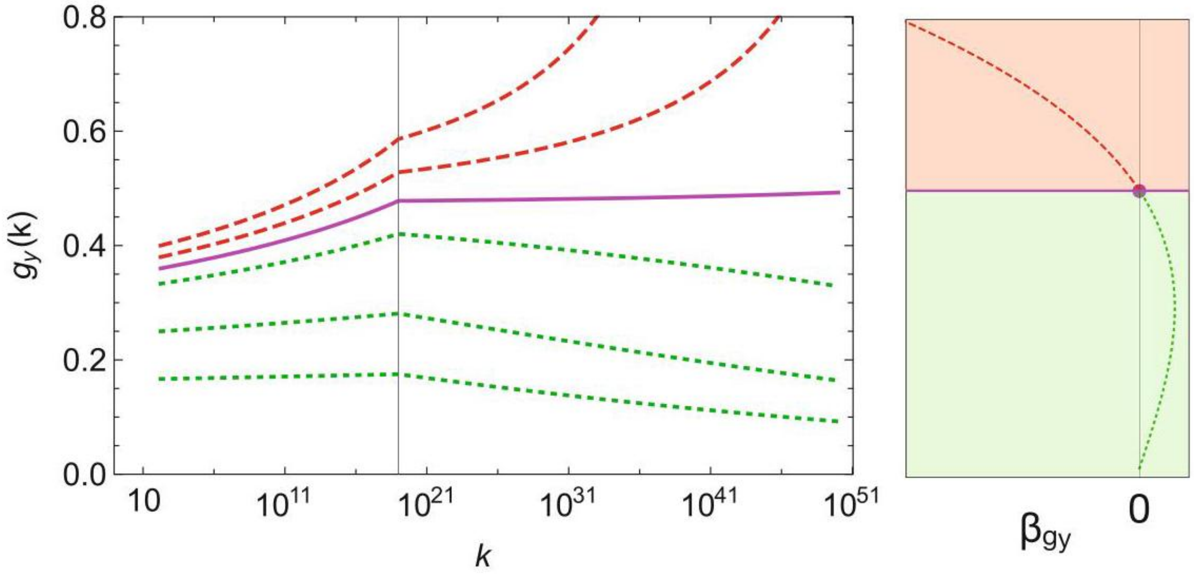


Fig. 12 We show the beta function for g_y (right panel, rotated), such that the gravity-dominated regime is the lower, green-shaded range of coupling values. The matter-dominated regime is the upper, red-shaded range of coupling values. Gravity and matter fluctuations balance out at an interacting fixed point (magenta). The resulting trajectories are shown in the left panel: in the matter-dominated regime, the triviality problem persists; in the gravity-dominated regime, trajectories emanate from an asymptotically free fixed point in the very far UV. A single trajectory is the asymptotically safe one, on which quantum scale symmetry holds at transplanckian scales. The fixed-point value is mapped to a unique value in the IR. For $f_{g_y} \approx 9.7 \cdot 10^{-3}$, the IR value corresponds to the measured IR value of the Abelian hypercharge coupling

图 12 我们展示了 g_y 的 β 函数 (右 panel，已旋转)，其中引力主导区是下方绿色阴影的耦合值范围，物质主导区是上方红色阴影的耦合值范围。引力与物质涨落在相互作用不动点 (洋红色) 达到平衡。得到的轨迹展示在左 panel: 物质主导区中平凡性问题仍然存在；引力主导区中，轨迹源自极紫外区的渐近自由不动点。其中只有一条轨迹是渐近安全的，在超普朗克能标上满足量子标度对称性。不动点值会被映射到红外区的唯一值。对 $f_{g_y} \approx 9.7 \cdot 10^{-3}$ 而言，该红外值对应阿贝尔超荷耦合测得的红外值。

Rephrased in a more technical manner, at this fixed point, g_y is an irrelevant direction with $\theta = -\partial\beta_{g_y}/\partial g_y|_{g_y=g_{y,*}} = -2f_{g_y} < 0$ and therefore does not contribute a free parameter. Hence, the IR value of the Abelian hypercharge following from this fixed point is a prediction of the UV completion.

用更专业的方式重述: 在该不动点处，g_y 是 $\theta = -\partial \beta_{g_y} / \partial g_y \big|_{g_y = g_{y,*}} = -2 f_{g_y} < 0$ 对应的无关方向，因此不会贡献自由参数。因此，由该不动点得到的阿贝尔超荷红外值是该紫外完备性的预言。

At the same time, the interacting fixed point generates an upper bound, above which the triviality problem cannot be avoided: trajectories for which $g_y(k = M_{\text{Planck}}) < g_{y,*}$ are those that come from an asymptotically free fixed point and are therefore UV complete. They reach IR values that lie below the value from the interacting fixed point. However, trajectories for which $g_y(k = M_{\text{Planck}}) > g_{y,*}$ diverge if followed further into the UV. They reach IR values that lie above the value from the interacting fixed point. In summary, the IR value of the gauge coupling is bounded from above by the prediction from asymptotic safety, if one wants to avoid the triviality problem.

同时, 相互作用不动点会产生一个上界, 超过该上界就无法避免平庸性问题: $g_y(k = M_{\text{Planck}}) < g_{y,*}$ 对应的轨迹源自渐近自由不动点，因此是紫外完备的，它们到达的红外值低于相互作用不动点给出的值。然而，若进一步向紫外延伸，$g_y(k = M_{\text{Planck}}) > g_{y,*}$ 对应的轨迹会发散，它们到达的红外值高于相互作用不动点给出的值。综上可知，若要避免平庸性问题，规范耦合的红外值会被渐近安全的预测限定在上界以内。

In consequence, asymptotic safety becomes testable: if the prediction for the coupling from the interacting fixed point (assuming this fixed point persists beyond the approximations it was seen in to date) is below the measured value, the triviality problem persists, and a UV completion of the SM with gravity is ruled out.

因此，渐近安全理论变得可检验: 如果相互作用不动点给出的耦合预测 (假设该不动点在目前已观测到的近似范围之外依然存在) 低于测量值，就说明平庸性问题依然存在，包含引力的标准模型紫外完备就被排除。

Intriguingly, the predicted value of the Abelian hypercharge comes out above the experimentally measured value and, within estimates of the systematic uncertainties of FRG computations, might even be in agreement with experimental observations [166,167]. Therefore, asymptotically safe quantum gravity might not only provide a UV completion for the Abelian gauge sector, but this UV completion might even be predictive, solving the long-standing riddle, why the fine-structure constant is 1/137 in the IR; see also [170,171].

有趣的是, 阿贝尔超荷的预测值高于实验测量值，而且在 FRG 计算的系统不确定性估计范围内，该预测甚至可能与实验观测一致 [166,167]。因此，渐近安全量子引力不仅可以为阿贝尔规范区提供紫外完备，这种紫外完备甚至还具有预测性，能够解决为何精细结构常数在红外区为 1/137 这个长期存在的谜题；另见 [170,171]。

The non-Abelian gauge sector of the SM with and without gravity has a simpler structure: for non-Abelian gauge couplings, the matter contribution $\beta_{g_i,1}$ is negative in the SM, giving rise to asymptotic freedom. The minimal coupling of gravity to matter is not sensitive to internal symmetries of the matter sector. Hence, the gravitational contribution f_{g_y} is also the gravitational contribution to the scale dependence of non-Abelian gauge couplings. Hence, asymptotic freedom for the strong coupling remains intact in the presence of asymptotically safe quantum gravity [107, 165]. The IR values of the two non-Abelian gauge couplings remain free parameters.

标准模型的非阿贝尔规范区无论是否包含引力，结构都更简单: 对于非阿贝尔规范耦合，标准模型中物质贡献 $\beta_{g_i,1}$ 为负，从而产生渐近自由。引力与物质的最小耦合不依赖物质区的内部对称性。因此，引力贡献 f_{g_y} 同时也是非阿贝尔规范耦合标度依赖性的引力贡献。因此，在渐近安全量子引力存在的情况下，强耦合的渐近自由依然保持成立 [107, 165]。两个非阿贝尔规范耦合的红外值仍然是自由参数。

Further Reading

拓展阅读

Chiral Symmetry Breaking with the Abelian Gauge Coupling

阿贝尔规范耦合的手征对称性破缺

Assuming the realization of the interacting fixed point for g_y , a fixed-point collision in induced chirally symmetric four-fermion interactions (see section ”Light Fermions”) may occur; see [131]. In this scenario, the nonvanishing fixed point value for g_y triggers a fixed-point collision in four-fermion interactions. Such a fixed-point collision is well studied in QCD, where it is associated with spontaneous breaking of chiral symmetry. If this effect were to occur in the asymptotically safe system, it would prevent the existence of light fermions, i.e., fermions with masses below the Planck scale. To prevent this, the value of the interacting fixed point has to be small enough to allow for a UV complete and chirally symmetric theory. This is only possible if the number of fermions in the system exceeds a critical number; see [131]. Hence, the interplay of quantum gravity and matter might put lower bounds on the number of fermions, in addition to the upper bounds discussed in section ”Light Fermions”.

假设 g_y 实现了相互作用不动点，就可能在手征对称的四费米子相互作用中诱发不动点碰撞 (参见“轻费米子”一节)，参见文献 [131]。在该机制中，g_y 非零的不动点值会触发四费米子相互作用中的不动点碰撞。这类不动点碰撞在量子色动力学中已有充分研究，它与手征对称性的自发破缺相关。如果该效应在渐近安全系统中发生，就会阻碍轻费米子 (即质量低于普朗克能标的费米子) 存在。为避免这一情况，相互作用不动点的取值必须足够小，才能使理论在紫外完备且保持手征对称。这只有当系统中的费米子数超过临界值时才能实现，参见文献 [131]。因此，除了“轻费米子”一节讨论的费米子数上限外，量子引力与物质的相互作用还可能给出费米子数的下限。

Subtleties in f_{g_y}

f_{g_y} 中的细微问题

The interpretation of f_{g_y} has some subtleties which we did not discuss above. For this discussion, it is useful to isolate the G_N dependence in f_{g_y} and write $f_{g_y} = G_N \tilde{f}_{g_y}$. Individual terms in beta functions are not necessarily physical and may therefore depend on the scheme. Indeed, in perturbative studies, it depends on the scheme, whether $\tilde{f}_{gy}$ vanishes or not; see, e.g.,[172-178]. From this, it was concluded that there is no gravitational contribution to the scale dependence of the Abelian gauge coupling. However, there is an important hidden assumption in these perturbative studies: they treat the gravitational coupling G_N as a constant which is finite. The gravitational contribution $f_{g_y} = G_N \tilde{f}_{g_y}$ vanishes, when $\tilde{f}_{g_y}$ vanishes and G_N is finite. However, when G_N diverges, one must be careful when evaluating f_{g_y} . In [169], it was shown that there are FRG regulators [179,180], for which $\tilde{f}_{g_y}$ also vanishes. In contrast to perturbative studies using dimensional regularization, the FRG regulator admits a smooth limit in which $\tilde{f}_{g_y}$ goes to zero as a function of a control parameter. As a function of the same control parameter, the fixed-point value $G_{N,*}$ diverges. We therefore find that f_{g_y} may indeed vanish for some schemes, when G_N is held fixed. However, in at least one of those schemes, $G_{N,*}$ diverges such that f_{g_y} remains finite while $\tilde{f}_{g_y}$ goes to zero.

f_{g_v} 的诠释存在一些我们上文未讨论的细微问题。在本次讨论中，分离出 f_{g_v} 中对 G_N 的依赖并写出 $f_{g_y} = G_N \tilde{f}_{g_y}$ 会更便于分析。β 函数中的单个项不一定是物理量，因此可能依赖于方案。实际上，在微扰研究中，$\tilde{f}_{gy}$ 是否不为零确实依赖于方案；参见例如文献 [172-178]。由此，有结论得出阿贝尔规范耦合的标度依赖不存在引力贡献。但这些微扰研究存在一个重要的隐含假设: 它们将引力耦合 G_N 当作有限的常数处理。当 $\tilde{f}_{g_y}$ 为零且 G_N 有限时，引力贡献 $f_{g_y} = G_N \tilde{f}_{g_y}$ 等于零。但当 G_N 发散时，计算 f_{g_y} 必须格外小心。文献 [169] 指出，存在满足 $\tilde{f}_{g_y}$ 同样为零的 FRG 调节器 [179,180]。与使用维数正规化的微扰研究不同，FRG 调节器允许一个光滑极限: 在该极限下，$\tilde{f}_{g_y}$ 作为调控参数的函数趋于零。作为同一调控参数的函数，不动点值 $G_{N,*}$ 发散。因此我们发现，当 G_N 固定时，f_{g_y} 确实可以在部分方案中为零。但至少在其中一类方案中，$G_{N,*}$ 发散，使得当 $\tilde{f}_{g_y}$ 趋于零时，f_{g_y} 仍保持有限。

Further, f_{g_y} evaluated at $G_N = G_{N,*}$ is a universal quantity, because it is a critical exponent at $g_{y,*} = 0$. As a universal quantity, it may not depend on the scheme (in practice, in approximations, it still does). It is therefore reassuring that even in schemes, in which $f_{g_y}(G_N) \to 0$, for the universal quantity, it holds that $f_{g_y}(G_{N,*}) \neq 0$.

此外，在 $G_N = G_{N,*}$ 处求值得到的 f_{g_y} 是一个普适量，因为它是 $g_{y,*} = 0$ 处的临界指数。作为普适量，它不应当依赖于方案 (实际在近似计算中它仍会依赖方案)。令人安心的是，即使在存在 $f_{g_y}(G_N) \to 0$ 的方案中，该普适量也满足 $f_{g_y}(G_{N,*}) \neq 0$.

We therefore conclude that in perturbation theory, when gravity does not assume a fixed point, it may be the case that gravitational contributions to beta functions vanish in some schemes. However, when gravity assumes a fixed point, and f_{g_y} is evaluated with the appropriate care, there is a nonzero gravitational contribution and f_{g_y} is a universal quantity.

因此我们得出结论: 在微扰论中，当引力不存在不动点时，引力对 β 函数的贡献可能确实会在部分方案中消失。但当引力存在不动点，且 f_{g_y} 经过恰当的小心计算后，引力贡献非零，且 f_{g_y} 是一个普适量。

Yukawa Couplings in the Standard Model

标准模型中的汤川耦合

Synopsis: Gravity can either screen or antiscreen a Yukawa coupling, depending on the gravitational fixed-point values. In the antiscreening case, the Yukawa coupling can become asymptotically free or safe, with an upper bound on its IR value. In the screening case, the Yukawa coupling is not UV complete.

概要: 引力既可以屏蔽也可以反屏蔽汤川耦合，具体取决于引力不动点的值。在反屏蔽情形下，汤川耦合可达到渐近自由或渐近安全，其红外值存在上界；在屏蔽情形下，汤川耦合不具备紫外完备性。

Based on this result, there is a mechanism that ties the quark masses to their charges: if gravity is anti-screening, and the Abelian gauge coupling is at its interacting fixed point, there is an interacting fixed point in the Yukawa sector, for which the up-type quarks and down-type quarks have different fixed-point values, because they are charged differently under the Abelian hypercharge.

基于这一结果，存在一个将夸克质量与其电荷联系起来的机制: 若引力为反屏蔽，且阿贝尔规范耦合处于相互作用不动点，则汤川 sector 会存在一个相互作用不动点；由于上型夸克和下型夸克在阿贝尔超荷下的电荷不同，二者在该不动点处取值不同。

For the third generation, this mechanism gives rise to a SM-like IR phenomenology, with bottom-quark mass and top-quark mass predicted at or in the vicinity, of their measured values.

对于第三代夸克，该机制产生了类似标准模型的红外唯象，预言的底夸克质量和顶夸克质量与实验测量值一致或非常接近。

For the full quark sector of the SM, mixing between flavors becomes important at highly transplanckian scales, and fixed points with nonzero Yukawa couplings no longer produce SM-like IR phenomenology. Instead, a fixed point which is made asymptotically free under the impact of gravity is available for all quark Yukawa couplings and CKM matrix elements, rendering the SM quark Yukawa sector UV complete.

对于标准模型的整个夸克 sector，味混合在远跨普朗克能标下变得重要，非零汤川耦合的不动点不再产生类似标准模型的红外唯象。此时，所有夸克汤川耦合和 CKM 矩阵元都可以存在一个在引力作用下达到渐近自由的不动点，使得标准模型的夸克汤川 sector 实现紫外完备。

In the SM, quark masses are generated by two mechanisms: first, by electroweak symmetry breaking in the Higgs-Yukawa sector, called the current mass, and second by chiral symmetry breaking in the strongly interacting phase of QCD, called the constituent mass. Lepton masses are generated only through electroweak symmetry breaking. The ratio of lepton masses and current quark masses to the Higgs vacuum expectation value is determined by Yukawa couplings - one for each quark flavor and lepton species. Schematically, this is the same as for a simple Yukawa system built out of a Dirac fermion and a real scalar, even though the SM is actually based on Weyl fermions and a complex Higgs scalar that is an SU(2) doublet. The Yukawa coupling $y\phi\bar{\psi}\psi$ between the Dirac fermion ψ , the corresponding antifermion $\bar{\psi}$, and the real scalar ϕ gives rise to a mass term, when the scalar develops a vacuum-expectation value $\langle\phi\rangle = v$ in the symmetry-broken phase. There, one can express the scalar field as excitations φ around its expectation value, leading to $y\phi\bar{\psi}\psi \rightarrow m\bar{\psi}\psi + y\varphi\bar{\psi}\psi$, where $m = yv$. In the SM, the IR values of the Yukawa couplings are therefore known, because the masses of all fermions have been measured. In addition, both ATLAS and CMS have measured the Yukawa couplings of the heaviest quarks [112-115] and the heaviest lepton [116, 117]. This motivates a study of the Yukawa sector coupled to asymptotically safe gravity, to find out, whether (i) the Yukawa sector is UV complete when gravity is present and (ii) whether the measured IR values can either be accommodated or even calculated from first principles.

在标准模型中，夸克质量由两种机制产生: 第一种是希格斯-汤川 sector 中的电弱对称性破缺，生成的质量称为流夸克质量；第二种是 QCD 强相互作用相中的手征对称性破缺，生成的质量称为组分夸克质量。轻子质量仅通过电弱对称性破缺产生。轻子质量、流夸克质量与希格斯真空期望值的比值由汤川耦合决定——每种夸克味、每种轻子各对应一个汤川耦合。大体来看，这和由狄拉克费米子与实标量场构成的简单汤川系统是一样的，尽管标准模型实际上是基于外尔费米子，以及作为 SU(2) 二重态的复希格斯标量。狄拉克费米子 ψ 、对应的反费米子 $\overline{\psi}$ 与实标量场 ϕ 之间的汤川耦合 $y\phi\overline{\psi}\psi$ ，会在标量场于对称破缺相产生真空期望值 $\langle\phi\rangle = v$ 时生成质量项。此时我们可以将标量场写为其真空期望值周围的激发 φ ，从而得到 $y\phi\overline{\psi}\psi \to m\overline{\psi}\psi + y\varphi\overline{\psi}\psi$ ，其中 $m = yv$ 。在标准模型中，汤川耦合的红外值是已知的，因为所有费米子的质量都已被测量。此外，ATLAS 和 CMS 都测量了最重夸克 [112-115] 与最重轻子 [116,117] 的汤川耦合。这推动了对耦合渐近安全引力的汤川 sector 的研究，以探究:(i) 存在引力时汤川 sector 是否是紫外完备的；(ii) 测量得到的红外值能否从第一性原理得到容纳，甚至被计算出来。

Simple Yukawa System

简单汤川系统

The structure of the gravity Yukawa system for the SM follows from the basic structure of a single Yukawa coupling and the gravitational effect on it: that structure is the same as for gauge couplings. Out of a competition between a screening matter contribution and an antiscreening gravity contribution, an asymptotically free fixed point arises. Trajectories that start from it reach a range of values in the IR, which is bounded from above by the prediction from an asymptotically safe fixed point. The difference between gauge and Yukawa sector is that this mechanism is only at work in a part of the gravitational parameter space, cf. Fig. 5.

标准模型引力汤川系统的结构源自单个汤川耦合的基本结构及其受到的引力效应: 该结构与规范耦合的结构相同。在物质的屏蔽贡献与引力的反屏蔽贡献的相互竞争中，产生了一个渐近自由不动点。从该不动点出发的轨迹在红外区得到一系列取值，这些取值的上界由渐近安全不动点的预测给出。规范 sector 与汤川 sector 的区别在于，该机制仅在引力参数空间的一部分区域生效，参见图 5。

For a simple Yukawa system as introduced above, the matter contribution in Eq. (24) is positive, $\beta_{y,1} > 0$, such that a simple Yukawa system has a Landau pole. The sign of the gravitational contribution f_y depends on the fixed-point value of the cosmological constant [95,97,110,111]; see also Fig. 5: for fixed-point values below a critical value Λ_{crit} , $f_y > 0$ holds, such that the Gaussian fixed point $y_* = 0$ is IR repulsive. Starting from this fixed point, finite values for the Yukawa couplings in the IR can be reached and the IR value is a free parameter. Just like in the Abelian gauge sector, $f_y > 0$ gives rise to a second, interacting fixed point, where the Yukawa coupling corresponds to an irrelevant, i.e., IR-repulsive direction. This fixed point hence is connected to a single predictive trajectory, where the IR value of the coupling is a prediction. This predictive trajectory produces an upper bound of the IR value of the Yukawa coupling, which can be reached from the Gaussian fixed point $y_* = 0$.

对于上文介绍的简单汤川系统，式 (24) 中的物质贡献为正，$\beta_{y,1} > 0$ ，因此简单汤川系统存在朗道极点。引力贡献 f_y 的符号取决于宇宙常数的不动点值 [95,97,110,111]，另参见图 5: 当不动点值低于临界值时，Λ_{crit} , $f_y > 0$ 成立，因此高斯不动点 $y_* = 0$ 是红外排斥的。从该不动点出发，可以得到红外区汤川耦合的有限值，且该红外值是一个自由参数。与阿贝尔规范 sector 一样，$f_y > 0$ 会产生第二个相互作用不动点，该不动点处汤川耦合对应一个无关方向，即红外排斥方向。因此这个不动点连接着一条唯一的预言轨迹，耦合的红外值是该轨迹给出的预言。这条预言轨迹为可从高斯不动点 $y_* = 0$ 得到的汤川耦合红外值给出了上界。

For $\Lambda > \Lambda_{\text{crit}}$, the gravitational contribution is screening, i.e., $f_y < 0$[95, 97 , 110,111]. This makes the Landau pole problem worse. Accordingly, if $\Lambda_* > \Lambda_{\text{crit}}$, the only possibility to achieve a UV-complete Yukawa sector is to set $y = 0$ at the Planck scale. Once y is set to zero, there is an additional global symmetry, namely, a chiral rotation for the fermion, $\psi \to e^{i\gamma_5 \alpha}\psi$. This symmetry protects the Yukawa coupling, such that it cannot be regenerated below the Planck scale. Accordingly, the case $f_y < 0$ results in a prediction, namely, of a vanishing Yukawa coupling. Thus, fixed-point values $\Lambda_* > \Lambda_{\text{crit}}$ are excluded from the viable parameter space for the gravitational fixed-point values, because vanishing Yukawa couplings are in contradiction with observations.

对于 $\Lambda > \Lambda_{\text{crit}}$ ，引力贡献为屏蔽，即 $f_y < 0$[95, 97 , 110,111]。这会加剧朗道极点问题。因此，如果 $\Lambda_* > \Lambda_{\text{crit}}$ ，要获得紫外完备的汤川 sector，唯一的方式是在普朗克尺度令 $y = 0$ 等于零。一旦 y 为零，系统就会多出一个整体对称性，即费米子的手征旋转 $\psi \to e^{i\gamma_5 \alpha}\psi$ 。该对称性会保护汤川耦合，使其无法在普朗克尺度以下重新生成。因此，情况 $f_y < 0$ 会给出汤川耦合为零的预言。因此，满足 $\Lambda_* > \Lambda_{\text{crit}}$ 的不动点值被排除在引力不动点值的可行参数空间之外，因为零汤川耦合与观测结果矛盾。

Calculations of the gravitational fixed-point values in the presence of a single Dirac fermion and real scalar (i.e., the fields that make up a simple Yukawa system) yield the result $\Lambda_* > \Lambda_{\text{crit}}$ [80, 82, 90, 96] . At larger number of fields, in particular, in the presence of all SM fields, different studies find differing results; however, e.g., [80, 96] (which rely on the background-field approximation) find that, once a third generation of SM fermions is present, the gravitational fixed-point value has moved to $\Lambda_* < \Lambda_{\text{crit}}$. Note that in [107] $\Lambda_* < \Lambda_{\text{crit}}$ is achieved in fluctuation computations by integrating out gravitational and matter fluctuations at slightly different scales.

对存在单个狄拉克费米子和实标量场 (即构成简单汤川系统的场) 时的引力不动点值计算得到结果 $\Lambda_* > \Lambda_{\text{crit}}$ [80, 82, 90, 96] 。当场数量更多，特别是存在所有标准模型场时，不同研究得到了不同结论；不过例如 [80, 96] (依赖背景场近似) 发现，一旦存在标准模型费米子的第三代，引力不动点值就会移动到 $\Lambda_* < \Lambda_{\text{crit}}$ 。注意在文献 [107] 中，通过对引力涨落和物质涨落在略有不同的尺度上积出，涨落计算也得到了 $\Lambda_* < \Lambda_{\text{crit}}$ 。

Further Reading

延伸阅读

Retrodicting the Top Mass

顶夸克质量的回溯预测

In [109], the gravity-generated interacting fixed point for the top Yukawa coupling allows to calculate the top-quark mass from first principles, yielding a value of about 171 GeV, which is, within the systematic uncertainties of the calculation, very well compatible with the experimental value of 172.8 GeV.

在文献 [109] 中，顶夸克汤川耦合由引力生成的相互作用不动点使得我们可以从第一性原理计算顶夸克质量，得到的结果约为 171 GeV；在该计算的系统误差范围内，这个结果与实验测得的 172.8 GeV 高度吻合。

Top-Bottom System

顶-底夸克系统

In the SM, the Yukawa couplings couple the right-handed $SU(2)$ -singlets to left-handed $SU(2)$ -doublets and the Higgs field. Nevertheless, the gravitational contribution is the same as for a real scalar coupling to a Dirac fermion and its antifermion. The underlying reason is that gravity is ”blind” to internal symmetries (in this case, the $SU(2)$). Therefore, the gravitational contribution to the top-quark Yukawa coupling y_t and the bottom-quark Yukawa coupling y_b is the same. Thus, one can, if $\Lambda_* < \Lambda_{\text{crit}}$, achieve asymptotic freedom for the two Yukawa couplings. Asymptotic safety for both Yukawa couplings is ruled out, because the fixed-point value for the top and bottom Yukawa is equal to each other. Thus, their IR values are also close to each other (they are not equal, because gauge field contributions to the two scale-dependences differ), and this contradicts experiment: the top quark is about forty times as heavy as the bottom quark.

在标准模型中，汤川耦合将右手 $SU(2)$ 单态与左手 $SU(2)$ 双态以及希格斯场耦合。尽管如此，引力贡献与实标量耦合到狄拉克费米子及其反费米子的情况相同。根本原因是引力对内部对称性 (在这里就是 $SU(2)$) 是“盲”的。因此，引力对顶夸克汤川耦合 y_t 和底夸克汤川耦合 y_b 的贡献是相同的。因此，若 $\Lambda_* < \Lambda_{\text{crit}}$ ，两个汤川耦合可以实现渐近自由。两个汤川耦合都满足渐近安全是不可能的，因为顶夸克和底夸克汤川的不动点值相等。因此它们的红外值也彼此接近 (但不相等，因为规范场对二者标度依赖性的贡献不同)，这与实验矛盾: 顶夸克质量约是底夸克的四十倍。

However, besides the gravitational contribution, there can also be a contribution from the Abelian gauge field. If g_y starts out at the asymptotically safe fixed point in section ”Gauge Couplings in the Standard Model”, then $y_{t*} \neq y_{b*}$ follows. This is because the top quark and the bottom quark do not have the same electric charge and thus also not the same $U(1)$ hypercharge. This can be seen from their beta functions, together with the beta function for the Abelian hypercharge. We provide these here in a truncation without higher-order couplings and to leading order in the couplings. We also do not show the non-Abelian gauge couplings in the beta functions below, because they have vanishing fixed-point values. Their effect is included, when one follows the RG flow from the fixed points down to the IR, as in Fig. 14.

但除了引力贡献外，阿贝尔规范场也可以提供贡献。如果 g_y 从“标准模型中的规范耦合”一节的渐近安全不动点出发，那么就有 $y_{t*} \neq y_{b*}$ 。这是因为顶夸克和底夸克的电荷不同，因此 $U(1)$ 超荷也不同。这可以从二者的 β 函数以及阿贝尔超荷的 β 函数中看出。我们在此在无高阶耦合的截断下、在耦合领头阶给出这些 β 函数。我们也没有在下方的 β 函数中写出非阿贝尔规范耦合，因为它们的不动点值为零。当人们遵循重整化群流从不中心点向下到红外，就像图 14 那样，它们的效应已经被包含在内了。

$$\beta_{y_t} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2 + \frac{3}{2}y_b^2 - \frac{17}{12}g_Y^2\right) - f_y y_t, \tag{30}$$

$$\beta_{y_b} = \frac{y_b}{16\pi^2}\left(\frac{9}{2}y_b^2 + \frac{3}{2}y_t^2 - \frac{5}{12}g_Y^2\right) - f_y y_b, \tag{31}$$

$$\beta_{g_Y} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g g_Y. \tag{32}$$

The system has an interacting fixed point, for which the fixed-point relation

该系统存在相互作用不动点，满足不动点关系

$$y_{t*}^2 - y_{b*}^2 = \frac{1}{3}g_{Y*}^2 \tag{33}$$

holds. This fixed-point relation distinguishes the fixed-point values for top and bottom Yukawa, as soon as a nonzero fixed-point value for the Abelian gauge coupling is realized. Then, the top-quark Yukawa coupling is also automatically much larger than the bottom-quark Yukawa coupling. The resulting beta functions for top and bottom are illustrated in Fig. 13. Taking into account the flow of all three gauge couplings of the SM, and choosing f_g and f_y appropriately, produces an IR phenomenology that is rather close to that of the SM, cf. Fig. 14.

成立。只要阿贝尔规范耦合存在非零不动点值，该不动点关系就能区分顶夸克和底夸克汤川的不动点值。此时，顶夸克汤川耦合也会自动远大于底夸克汤川耦合。由此得到的顶夸克和底夸克的 β 函数如图 13 所示。计入标准模型全部三种规范耦合的跑动，并适当选取 f_g 和 f_y ，得到的红外唯象与标准模型十分接近，参见图 14。

This mechanism is remarkable, because it links the charge ratio of the two quarks to its mass ratio. In fact, other charge ratios are incompatible with the measured masses of top and bottom quark, even if arbitrary values of f_y and f_g are considered.

这一机制非常引人注目，因为它将两种夸克的电荷比与质量比联系了起来。实际上，即使考虑任意的 f_y 和 f_g 取值，其他电荷比都与顶夸克、底夸克的测量质量不兼容。

In calculations based on Eqs. (30)-(32), full agreement with the measured values cannot be reached, and the top quark is $\sim 5-10$GeV too heavy (depending on which approximation is used), cf. [170, 181]. However, these calculations come with significant systematic uncertainties, e.g., by neglecting further, higher-order interactions.

基于式 (30)-(32) 的计算无法得到与测量值完全一致的结果，顶夸克 ~ 5 – 10GeV 过重 (取决于所用近似)，参见文献 [170, 181]。但这些计算存在显著的系统不确定性，例如忽略了更多高阶相互作用。

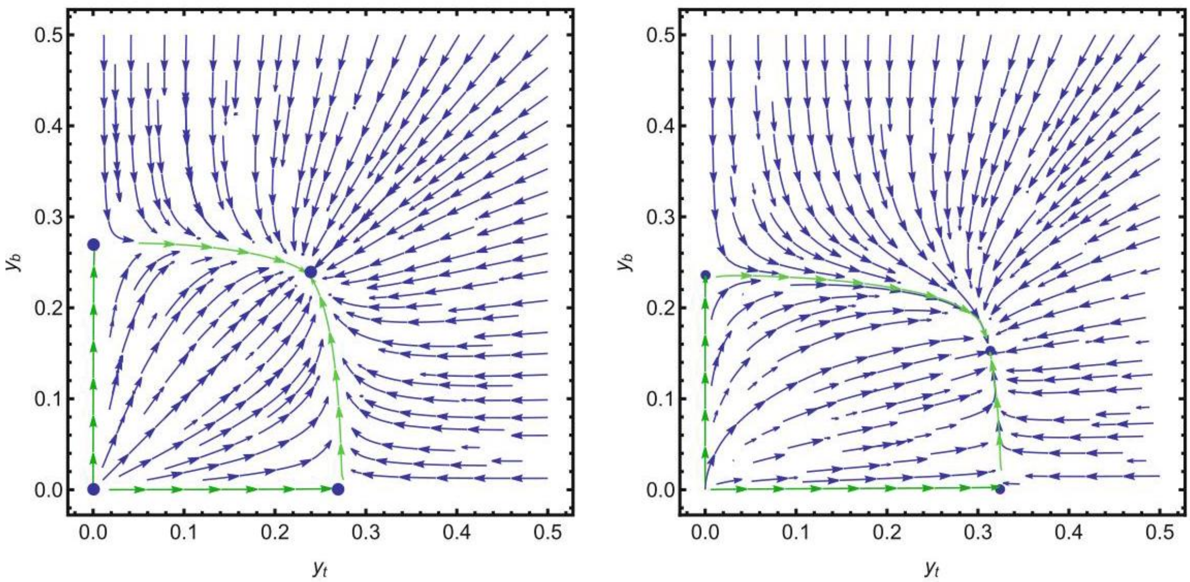


Fig. 13 We show the RG flow toward the IR in the plane spanned by y_t and y_b . In the absence of the Abelian gauge field (left panel), there is complete symmetry between y_t and y_b . In particular, the fully interacting fixed point, which attracts all trajectories, lies at $y_{t*} = y_{b*}$, resulting in a prediction of $y_t(M_{\text{Planck}}) = y_b(M_{\text{Planck}})$, which cannot result in viable IR phenomenology. In the presence of an Abelian gauge coupling (right panel), the symmetry is broken. For purposes of illustration, we have chosen a large f_y . For $f_y = 1.188 \cdot 10^{-4}$, as in [170], the fully interacting fixed point lies at $y_{b*} \ll y_{t*}$, very close to the fixed point at $y_{t*} \neq 0, y_{b*} = 0$

图 13 我们展示了红外方向在 y_t 与 y_b 张成的平面内的重整化群流。不存在阿贝尔规范场时 (左图)，y_t 与 y_b 之间存在完整对称性。具体来说，吸引所有轨迹的完全相互作用不动点位于 $y_{t*} = y_{b*}$，给出的预言为 $y_t(M_{\text{Planck}}) = y_b(M_{\text{Planck}})$，无法得到可行的红外唯象学。存在阿贝尔规范耦合时 (右图)，对称性发生破缺。为了便于演示，我们选取了一个较大的 f_y。对于文献 [170] 中的 $f_y = 1.188 \cdot 10^{-4}$，完全相互作用不动点位于 $y_{b*} \ll y_{t*}$，非常接近位于 $y_{t*} \neq 0, y_{b*} = 0$ 的不动点

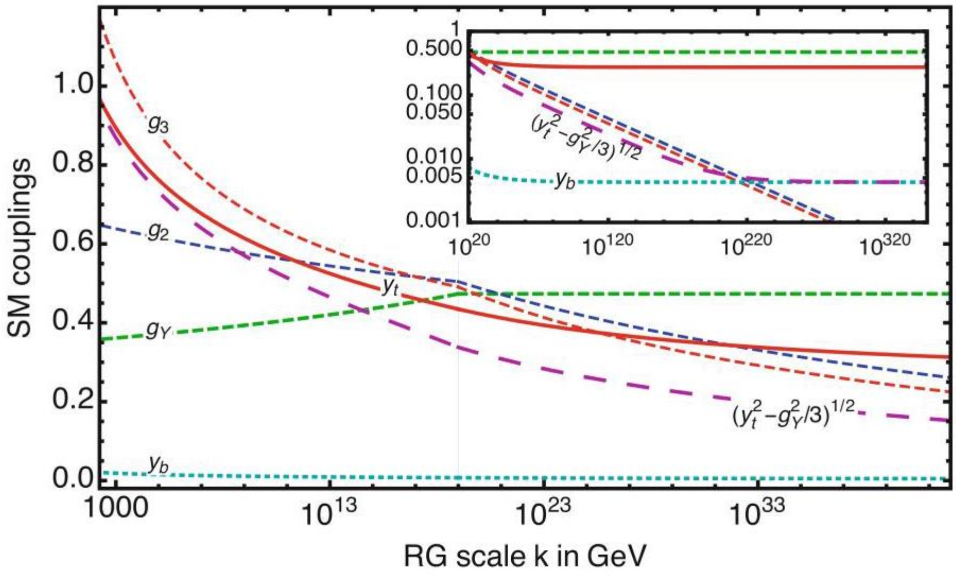


Fig. 14 We show the scale dependence of the top-bottom gauge system, with all three gauge couplings and the two Yukawa couplings, cf. [170]. Above the Planck scale, y_t, y_b , and g_Y start out at an interacting fixed point that satisfies relation Eq. (33). $f_g = 9.7 \cdot 10^{-3}$ and $f_y = 1.188 \cdot 10^{-4}$ are chosen such that the bottom-quark Yukawa and Abelian gauge coupling are predicted to agree with their measured value. The resulting top-quark Yukawa coupling is somewhat higher than in the SM

图 14 我们展示了顶-底夸克规范系统的标度依赖性，包含全部三个规范耦合和两个汤川耦合，参见文献 [170]。普朗克标度以上，y_t, y_b 与 g_Y 从满足式 (33) 关系的相互作用不动点出发。我们选取 $f_g = 9.7 \cdot 10^{-3}$ 和 $f_y = 1.188 \cdot 10^{-4}$ ，使得底夸克汤川耦合与阿贝尔规范耦合的预言和测量值一致。得到的顶夸克汤川耦合比标准模型中的结果稍大

Further, the Yukawa couplings of the other generations cannot be neglected at very high scales, contrary to what one may first think. This is because, although the other Yukawa couplings themselves are very small compared to y_t , the three-generation quark system includes CKM mixing. At very high energies, the CKM matrix elements are scale dependent, triggering a deviation of the fixed-point structure from Eq. (33). Therefore, we consider the full quark sector of the SM next.

此外，和初想不同，其他代的汤川耦合在极高能标下不能忽略。这是因为，尽管其他汤川耦合本身比 y_t 小得多，三代夸克系统包含 CKM 混合。在极高能下，CKM 矩阵元是标度依赖的，会导致不动点结构偏离式 (33)。因此我们接下来考察标准模型的完整夸克 sector。

Quark Sector of the SM

标准模型的夸克领域

In the SM, the quark Yukawa sector contains ten beta functions: six for the Yukawa couplings and four for the CKM matrix elements. The CKM matrix describes mixing in the quark sector, i.e., the electroweak interaction can change the flavors. Because it is unitary, the CKM matrix contains four physical parameters, with gravity-independent beta functions. Note that the independence of the CKM matrix from gravity contributions can be shown from the flavor universality of gravity, i.e., the ”blindness” of gravity to internal symmetries.

在标准模型中，夸克汤川领域包含 10 个 β 函数:6 个对应汤川耦合，4 个对应 CKM 矩阵元。CKM 矩阵描述夸克领域的混合，即电弱相互作用可以改变夸克味。由于 CKM 矩阵是幺正矩阵，它包含 4 个独立物理参数，且这些参数的 β 函数与引力无关。需要注意的是，CKM 矩阵不受引力贡献这一独立性可以由引力的味普适性，即引力对内部对称性“不可见”推导得出。

In [181] (see also [182]), the resulting complexity of the analysis was dealt with by making an assumption about the fixed-point structure, namely, that the CKM matrix elements assume fixed-point values which are independent of the Yukawa fixed-point values. The fixed-point conditions then factorize, and a fixed-point search for the CKM matrix elements can be conducted first.

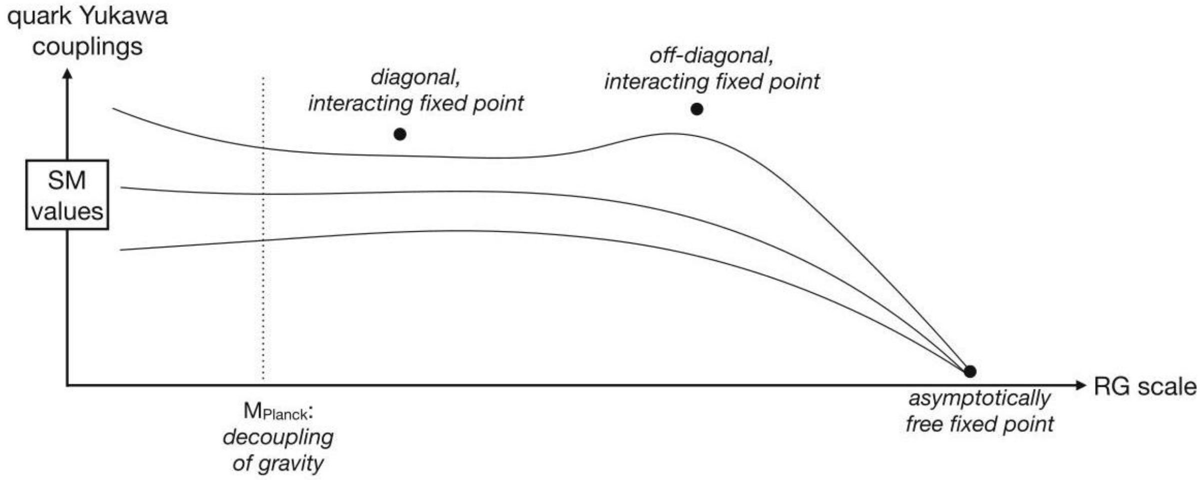
文献 [181](另见 [182]) 中，作者通过对不动点结构做假设来处理分析带来的复杂性: 假设 CKM 矩阵元的不动点值独立于汤川耦合的不动点值。这样一来不动点条件可以因式分解，因此可以先对 CKM 矩阵元搜索不动点。

Thereby, one obtains several simple fixed-point configurations for the CKM matrix, which have zeros or ones as the only entries. Two of those fixed-point solutions are phenomenologically important: it was already observed in [183] that a diagonal CKM matrix (which is close to the actual measured values) is an IR fixed point, because it has three IR attractive directions. This fixed point is approached in the IR, starting from another fixed point, namely, an off-diagonal CKM matrix with four IR repulsive directions. The corresponding flow is very slow (even on logarithmic scales, the CKM matrix elements are essentially constant); therefore, the transition from the off-diagonal to the near-diagonal configuration occurs at highly transplanckian scales.

由此，我们得到了数种简单的 CKM 矩阵不动点构型，这些构型的矩阵元仅为零或一。其中两种不动点解在唯象上十分重要: 文献 [183] 早已指出，对角 CKM 矩阵 (与实际测量值十分接近) 是红外不动点，因为它拥有三个红外吸引方向。从另一个不动点——即带有四个红外排斥方向的非对角 CKM 矩阵——出发，系统流向红外时会趋近这个对角不动点。对应的演化流非常缓慢 (即便在对数尺度下，CKM 矩阵元也基本保持恒定)，因此从非对角构型到近对角构型的转变发生在远超普朗克能标的尺度上。

In a second step, one can analyze the consequences for the Yukawa system. Because the CKM matrix elements enter the beta functions for the Yukawa couplings, those beta functions change, when the CKM matrix changes from an off-diagonal to a near-diagonal configuration. Therefore, in the very far UV, where the CKM matrix is off-diagonal, the fixed-point values for the Yukawa couplings are modified compared to an analysis without flavor mixing, as in [170]. It turns out that among the many fixed points that the beta functions have, only the asymptotically free one is phenomenologically relevant. It can be achieved if $f_y > -2.2 \cdot 10^{-4}$. Interacting fixed points, most importantly one at which $y_{t*} \neq 0$, remain important, because, starting from the asymptotically free fixed point in the very far UV, the system approaches such an interacting fixed point at intermediate scales; see the schematic illustration in Fig. 15.

第二步，我们可以分析这对汤川系统的影响。由于 CKK 矩阵元会进入汤川耦合的 β 函数，当 CKM 矩阵从非对角构型转变为近对角构型时，β 函数也会发生变化。因此，在极紫外区域，CKM 矩阵仍为非对角，此时汤川耦合的不动点值会不同于 [170] 中不考虑味混合的分析结果。结果表明，在 β 函数拥有的众多不动点中，只有渐近自由不动点具有唯象相关性，它可以在 $f_y > -2.2 \cdot 10^{-4}$ 的条件下实现。而相互作用不动点 (其中最重要的是满足 $y_{t*} \neq 0$ 的那个不动点) 仍然十分重要，因为从极紫外的渐近自由不动点出发，系统在中间能标会趋近这类相互作用不动点；示意图见图 15。



$$V_{\mathrm{CKM}}^{\mathrm{IR}} \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longleftarrow V_{\mathrm{CKM}}^{\mathrm{UV}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Fig. 15 Under the impact of asymptotically safe gravity, the quark Yukawa couplings start out at an asymptotically free fixed point in the very deep UV, at which the CKM matrix is off-diagonal. They are attracted toward an interacting fixed point, due to its irrelevant directions. When the CKM matrix elements transition toward a near-diagonal configuration, the fixed-point values at the interacting fixed point for the Yukawa couplings change. Over a large range of scales, that fixed point determines the properties of the Yukawa system. At the Planck scale, gravity decouples dynamically, and the flow of the quark Yukawa sector is exactly that of the SM, cf. [181] for more details

图 15 在渐近安全引力的作用下, 夸克汤川耦合的初始态是深紫外区域的渐近自由不动点, 此时 CKM 矩阵为非对角。之后，由于相互作用不动点的无关方向，耦合被吸引向该不动点。当 CKM 矩阵元向近对角构型转变时, 相互作用不动点处汤川耦合的不动点值也会发生改变。在很大的能标范围内, 都是这个不动点决定汤川系统的性质。到普朗克能标后，引力动力学退耦，夸克汤川领域的演化流就和标准模型完全一致，更多细节参见 [181]。

In summary, asymptotically safe gravity may UV complete the quark Yukawa sector of the SM. It may even have predictive power: if one assumes that asymptotic safety holds above the Planck scale, but not to arbitrarily high scales (e.g., because of a more fundamental UV completion), the mechanism discussed in section "Top-Bottom System" may link quark masses to their charges. Only if one insists that the RG flow should continue to make sense at scales as high as $k \approx 10^{1000}$GeV , does the flow of the CKM matrix matter, triggering an approach toward the asymptotically free fixed point (if one follows the RG flow in the reverse direction, i.e., toward the UV).

综上, 渐近安全引力可以为标准模型的夸克汤川领域提供紫外完备性, 它甚至具备预言能力: 如果我们假设渐近安全在普朗克能标以上成立，但不成立到任意高能标 (例如存在更基础的紫外完备性), 那么“顶夸克-底夸克系统”一节讨论的机制就可以将夸克质量和其电荷联系起来。只有当人们坚持认为重整化群流在高达 $k \approx 10^{1000}$GeV 的能标下仍然成立时，CKM 矩阵的流才会发挥作用，驱动系统趋近渐近自由不动点 (如果我们逆着重整化群流向紫外追踪的话)。

Several open questions currently remain, including (i) a study including the full lepton sector of the SM; (ii) a search for fixed points, for which the factorization hypothesis between CKM matrix elements and Yukawa couplings is given up; and (iii) the inclusion of higher-order effects in the beta functions, coming from additional interactions beyond the truncations considered to date.

目前该方向仍有多个开放问题，包括:(i) 纳入标准模型完整轻子领域的研究；(ii) 放弃 CKM 矩阵元与汤川耦合的因式分解假设，重新搜索不动点；(iii) 在 β 函数中纳入高阶效应，即纳入目前截断所忽略的额外相互作用。

Further Reading

扩展阅读

Learning About Dark Sectors from Predictions for the SM

从标准模型的预测研究暗区

All matter gravitates. This is also true for dark matter, such that the scale dependence of the gravitational couplings is affected by all visible and dark matter. Hence, dark matter influences the fixed-point values of the Newton coupling and the cosmological constant. These in turn enter the gravitational contribution to the scale dependence of SM couplings and hence determine the interacting fixed-point value for the Abelian gauge coupling and the Yukawa couplings. On the one hand, this changes the prediction for the low-energy values arising from the interacting fixed point. At the same time, the presence of dark matter lowers the upper bound on the low-energy couplings, which can be reached by the free fixed points. Hence, if too many dark-matter fields are present, the lower bound might drop below the experimentally observed value, thereby ruling such dark-matter models out. Thus, the predicted low-energy value of SM couplings might put constraints on the number of dark-matter fields. In [109], a proof of principle for this idea was given, because it was shown that the upper bound on the top-quark mass depends on the presence of additional matter fields. According to that analysis, if three right-handed neutrinos and an axion are added to the SM, the upper bound increases, which results in viable IR phenomenology (and implies an asymptotically free fixed point for the top Yukawa).

所有物质都参与引力相互作用，暗物质也不例外，因此引力耦合的标度依赖性会受到所有可见物质与暗物质的影响。因此，暗物质会影响牛顿耦合与宇宙常数的不动点值，而二者又会进一步参与引力对标准模型耦合标度依赖性的贡献，进而决定阿贝尔规范耦合与汤川耦合的相互作用不动点值。一方面，这会改变由相互作用不动点得出的低能值预测；同时，暗物质的存在会降低低能耦合的上限，而自由不动点可以达到这一区间。因此，如果存在过多暗物质场，该下限就会降到实验观测值以下，从而排除这类暗物质模型。由此，标准模型耦合的预测低能值可以对暗物质场的数量给出约束。文献 [109] 已经对这一思路给出了原理验证，研究表明顶夸克质量的上限依赖于额外物质场的存在。根据该分析，若在标准模型中加入三个右手中微子和一个轴子，顶夸克质量上限会提高，从而得到可行的红外唯象学 (且暗示顶汤川耦合存在渐近自由不动点)。

Higgs Quartic Coupling in the Standard Model

标准模型中的希格斯四次耦合

Synopsis: Gravity screens the Higgs quartic interaction, such that the ratio of the Higgs mass to the Higgs vacuum expectation value is a prediction from asymptotically safe quantum gravity, dating back to before the measurement of the Higgs mass at the LHC.

概要: 引力会屏蔽希格斯四次相互作用，因此希格斯质量与希格斯真空期望值的比值是渐近安全量子引力给出的预言，该预言早在 LHC 测量希格斯质量之前就已提出。

The value for this prediction depends on the gauge and Yukawa couplings in the SM, in particular the Abelian gauge coupling and the top Yukawa coupling. If both are asymptotically free, the Higgs mass is predicted to be only a few GeV above the experimental value (at least if the central value for the top-quark mass is assumed). If both are asymptotically safe, the Higgs mass comes out larger than that but can be lowered by a BSM Higgs portal coupling to a dark sector.

该预言的数值取决于标准模型中的规范耦合与汤川耦合, 具体而言是阿贝尔规范耦合和顶夸克汤川耦合。如果两者都是渐近自由的，预言出的希格斯质量仅比实验值高几 GeV(至少在采用顶夸克质量的中心值时是如此)。如果两者都是渐近安全的，得到的希格斯质量会更大，但可以通过超出标准模型的希格斯门户耦合到暗区来降低质量。

The beta function for the Higgs quartic coupling λ_H is given by

希格斯四次耦合 λ_H 的 β 函数由下式给出

$$\beta_{\lambda_{\mathrm{H}}} = -f_s\lambda_{\mathrm{H}} + \frac{1}{16\pi^2}\left(-6y_t^4 + \frac{3}{8}\left(2g_2^4 + \left(g_2^2 + \frac{5}{3}g_Y^2\right)^2\right)\right)$$

$$+\frac{1}{16\pi^2}\lambda_{\mathrm{H}}\left(12y_t^2 - 9g_2^2 - 5g_Y^2\right) + \frac{3}{2\pi^2}\lambda_{\mathrm{H}}^2, \tag{34}$$

with the top Yukawa coupling y_t , the Abelian hypercharge coupling g_Y , the $\mathrm{SU}(2)_L$ gauge coupling g_2 , and the gravitational contribution f_s . All other Yukawa couplings also contribute in principle but are in practice negligible, because all other fermions are much lighter than the top quark. In [184], the following idea was developed: if the gauge couplings and the top Yukawa coupling are asymptotically free under the impact of quantum gravity and $f_s < 0$, then Eq. (34) has the fixed-point solution $\lambda_{H*} = 0$. This fixed point is IR attractive, i.e., quantum-gravity fluctuations ensure that $\lambda_H\,(k = M_{\text{Planck}}) = 0$. It is one of the astonishing results of the LHC that $\lambda_H\,(k = M_{\text{Planck}}) \approx 0$ is what needs to be realized to obtain the measured Higgs mass, assuming that there is no new physics between the Planck scale and LHC energies. Explaining this special value is indeed one of the key challenges for particle physics at the moment.

其中包含顶夸克汤川耦合 y_t 、阿贝尔超荷耦合 g_Y 、$\mathrm{SU}(2)_L$ 规范耦合 g_2 ，以及引力贡献 f_s 。原则上所有其他汤川耦合也有贡献，但实际上都可以忽略，因为其他所有费米子都远轻于顶夸克。文献 [184] 提出了如下观点: 如果在量子引力和 $f_s < 0$ 的作用下，规范耦合与顶夸克汤川耦合都是渐近自由的，那么式 (34) 存在不动点解 $\lambda_{H*} = 0$ 。该不动点是红外吸引的，即量子引力涨落保证了 $\lambda_H\,(k = M_{\text{Planck}}) = 0$ 。如果假设普朗克能标到 LHC 能标之间不存在新物理，那么要得到测量得到的希格斯质量，就需要实现 $\lambda_H\,(k = M_{\text{Planck}}) \approx 0$ ，这确实是 LHC 得出的惊人结果之一。解释这个特殊值的确是当前粒子物理学的核心挑战之一。

Below the Planck scale, the Higgs quartic interaction is regenerated by gauge and top-quark fluctuations, which enter through the terms y_t^4 and g_i^4 in Eq. (34). The vanishing Planck-scale value of λ_{H} is thereby mapped onto a unique value at the electroweak scale, where it determines the ratio of Higgs mass M_{Higgs} and Higgs vacuum expectation value $v_{\text{Higgs}} = 246\text{GeV}$:

普朗克能标以下，希格斯四次相互作用会被规范涨落和顶夸克涨落重新生成，这一过程通过式 (34) 中的 y_t^4 项和 g_i^4 项进入。λ_{H} 在普朗克能标处的零值会被映射到电弱能标处的唯一确定值，该值决定了希格斯质量 M_{Higgs} 和希格斯真空期望值 $v_{\text{Higgs}} = 246\text{GeV}$ 的比值:

$$\lambda_H\,(k_{\mathrm{IR}}) = \frac{1}{2}\left(\frac{M_{\text{Higgs}}}{v_{\text{Higgs}}}\right)^2 . \tag{35}$$

The map depends sensitively on the top Yukawa coupling [185], which is only known with a significant systematic uncertainty. Thereby, a Higgs mass of $M_{\text{Higgs}} = 129\text{GeV}$ comes out for a top mass of $M_t = 172.9\text{GeV}$, but a top mass of $M_t = 170.9\text{GeV}$ is not ruled out, which means that the measured value $M_{\text{Higgs}} = 126\text{GeV}$ could be compatible with $\lambda_H\,(k = M_{\text{Planck}}) = 0$.

这一映射对顶夸克汤川耦合非常敏感 [185], 而顶夸克汤川耦合目前仍存在较大的系统不确定性。因此, 当顶夸克质量为 $M_t = 172.9\text{GeV}$ 时, 得到的希格斯质量为 $M_{\text{Higgs}} = 129\text{GeV}$; 但 $M_t = 170.9\text{GeV}$ 的顶夸克质量也并未被排除, 这意味着测量值 $M_{\text{Higgs}} = 126\text{GeV}$ 可以与 $\lambda_H\,(k = M_{\text{Planck}}) = 0$ 相容。

Thus, the idea in [184] led to a successful prediction of the Higgs mass in the vicinity of the measured value, several years prior to the experimental discovery of the Higgs particle at the LHC [186, 187]. This distinguishes the Higgs sector from the other sectors of the SM, where asymptotic safety may also allow to calculate masses and couplings from first principles, however, only after they have been measured, i.e., as "postdictions," not genuine predictions.

因此，文献 [184] 中的观点在 LHC 实验发现希格斯粒子的数年之前 [186, 187]，就成功预言了希格斯质量处于测量值附近。这一点将希格斯能区与标准模型的其他能区区分开: 其他能区中渐近安全或许也允许从第一性原理计算质量和耦合，但这些计算都是在它们被测量之后完成的，即属于“后言”，而非真正的预言。

There is a second difference to the prediction in the gauge and Yukawa sector: there, the IR values of the couplings depend sensitively on the values of f_g and f_y . In contrast, the prediction for the Higgs mass only depends on the sign of f_s . This is because the fixed point for λ_H is always the noninteracting one, $\lambda_{H*} = 0$, independent of the value of f_s . As long as $f_s < 0$, this fixed point is IR attractive and a prediction follows.

这与规范和汤川领域的预测存在第二个差异: 在那些领域，耦合的红外值对 f_g 和 f_y 的值十分敏感。与之相对，希格斯质量的预测仅取决于 f_s 的符号。这是因为 λ_H 的不动点始终是非相互作用不动点 $\lambda_{H*} = 0$ ，与 f_s 的取值无关。只要满足 $f_s < 0$ ，该不动点就是红外吸引的，由此便可得出一个预测。

Following [81, 83, 95, 97, 106, 109, 126, 188, 189] and even predating [78] the idea for the prediction of the Higgs mass [184], evidence for $f_s < 0$ has accumulated (with many papers working in the opposite sign convention with $\beta_{\lambda_H}\big|_{\text{gravity}} = f_s\lambda_H$).

沿用文献 [81, 83, 95, 97, 106, 109, 126, 188, 189] 的研究，甚至早于预测希格斯质量的文献 [184] 与文献 [78]，$f_s < 0$ 的相关证据已不断积累 (许多研究采用相反的符号约定，对应 $\beta_{\lambda_H}\big|_{\text{gravity}} = f_s\lambda_H$)。

As a second possibility for a UV-complete Higgs sector, the Abelian hypercharge and top Yukawa coupling may assume an interacting fixed point, in turn inducing an interacting fixed point for the Higgs quartic coupling:

作为希格斯场可紫外完备的第二种可能，阿贝尔超荷与顶夸克汤川耦合可以处于相互作用不动点，进而为希格斯四次耦合诱导出一个相互作用不动点:

$$\lambda_{\mathrm{H}*} = \frac{5}{48}g_{\mathrm{y},*}^2 - \frac{1}{4}y_{t*}^2 + \frac{\pi^2}{3}f_s + \frac{1}{48}\sqrt{\left(12y_{t*}^2 - 5g_{\mathrm{y},*}^2 - 16\pi^2 f_s\right)^2 + 576y_{t*}^4 - 100g_{\mathrm{y},*}^4}.$$

(36)

The fixed-point value now depends on the value of f_s , not just its sign, and also depends on gauge and Yukawa coupling. If they assume fixed-point values which result in the measured values in the IR, the Higgs mass comes out larger than 129 GeV. This rules out such an interacting fixed point in the SM with gravity. New physics is thus required in this scenario; see below.

此时不动点的值不仅取决于 f_s 的符号，还取决于 f_s 的具体取值，同时也与规范耦合和汤川耦合相关。若这些耦合处于能在红外区域得到测量值的不动点，那么得出的希格斯质量会大于 125 GeV。这就排除了标准模型中包含引力的这类相互作用不动点，因此该情形要求存在新物理，详见下文。

Further Reading

扩展阅读

Higgs Mass Prediction with Dark Sector

含暗区的希格斯质量预言

In [190], it was proposed that adding a Higgs portal, i.e., a coupling between the Higgs field and a dark scalar field, to an interacting dark sector with scalar and fermions could simultaneously solve two problems: first, it could provide a particle candidate to explain the observed dark-matter relic density. Second, it could shift the predicted Higgs mass toward lower values, starting from the fixed point at which top Yukawa and Abelian gauge coupling are nonzero. This would place a truly asymptotically safe UV completion of the SM with gravity and dark matter, with a high predictive power, within reach. The study in [190] is done within a toy model of the full SM (containing a real scalar as the Higgs and a Dirac fermion as the top quark, but no gauge fields), with the extension to the full SM an obvious and important open question.

文献 [190] 提出，在包含标量和费米子的相互作用暗区中引入希格斯门户 (即希格斯场与暗标量场之间的耦合)，可以同时解决两个问题: 其一，它能提供一个粒子候选者来解释观测到的暗物质残余密度；其二，从顶夸克汤川耦合与阿贝尔规范耦合均不为零的不动点出发，它能将预言的希格斯质量往更低值偏移。这使得结合引力与暗物质、具备高预言能力的标准模型真正渐近安全紫外完备理论有望被得到。文献 [190] 的研究是在完整标准模型的玩具模型中完成的 (该模型中实标量为希格斯，狄拉克费米子为顶夸克，但不包含规范场)，推广到完整标准模型是一个明确且重要的开放问题。

Similarly, in [191], it was found that a new massive Z' boson, corresponding to a gauged $U(1)_{B-L}$ symmetry, lowers the predicted value of the Higgs mass, while a sterile quark axion only has little impact on the prediction.

类似地，文献 [191] 发现，对应于整体规范 $U(1)_{B-L}$ 对称性的新的大质量 Z' 玻色子会降低希格斯质量的预言值，而惰性夸克轴子对预言结果几乎没有影响。

Resurgence Mechanism and Scalar Mass Parameter

复活机制与标量质量参数

In [184], the ratio of Higgs mass to electroweak scale is predicted, but not the electroweak scale itself. This is because the Higgs mass parameter, i.e., the quadratic term in an expansion of the potential about vanishing field value, is a free parameter, i.e., assumed to be RG relevant. In [192], it was suggested that this could change, consistent with results in [78, 126], which confirm that quantum gravity contributes negatively to the corresponding critical exponent. Thus, if gravitational fluctuations are large enough, the Higgs mass parameter becomes irrelevant. If it does so at vanishing fixed-point value, the resulting low-energy prediction is a vanishing electroweak scale. However, if the asymptotically safe gravity-matter fixed point has nonvanishing gauge and/or Yukawa couplings, these shift this fixed-point value away from zero. Whether or not this may lead to a scenario in which the electroweak scale is predicted at the right value is currently an open question. It should be stressed, though, that the required strength of gravitational fluctuations is large and not compatible with weak-gravity bounds; see section ”Step 2: The Symmetry-Preserving Interactions Which Are Generated by Gravity Have an Asymptotically Safe Fixed Point”.

在文献 [184] 中，预测了希格斯质量与电弱标度的比值，但未预测电弱标度本身。这是因为希格斯质量参数 (即势在场值为零处展开中的二次项) 是自由参数，被认定为 RG 相关量。在文献 [192] 中，研究者提出情况可能并非如此，这与文献 [78, 126] 的结果一致——这些结果证实量子引力对对应临界指数有负贡献。因此，如果引力涨落足够大，希格斯质量参数会变为无关量。若它在固定点值为零时变为无关量，得到的低能预测就是电弱标度为零。然而，如果渐近安全引力-物质固定点存在非零的规范和/或汤川耦合，这些耦合会将固定点值偏离零。这是否能形成电弱标度被预测为正确值的情景，目前仍是一个开放性问题。但需要强调的是，引力涨落所需的强度很大，与弱引力边界不兼容；参见章节“步骤 2: 引力生成的对称保持相互作用具有渐近安全固定点”。

Higgs Inflation

希格斯暴涨

Higgs inflation [193] is based on a nonminimal coupling ξ between the Higgs scalar and the curvature scalar. It could explain inflation without the need for extra fields beyond the SM. A change to the Einstein frame, i.e., a conformal transformation of the metric that removes the nonminimal coupling, produces a potential that is appropriate for inflation for suitable values of the couplings. This model is attractive due to its predictive power, because it does not require any BSM fields and contains only one free parameter, namely, the nonminimal coupling. In [194], it was found that the nonminimal coupling is predicted in asymptotic safety, at least for those values of G and Λ , for which Yukawa couplings can be nonzero. It turns out that the predicted ratio λ_4/ξ^2 between the Higgs quartic coupling and the nonminimal coupling is much too large to be compatible with CMB data. This result, if confirmed in extended truncations, rules out Higgs inflation in asymptotically safe gravity. The same conclusion was achieved already in [189], based on a study of the Higgs potential at large field values.

希格斯暴涨 [193] 基于希格斯标量与曲率标量之间的非最小耦合 ξ 。它无需超出标准模型的额外场就能解释暴涨过程。通过变换到爱因斯坦框架，即对度规做共形变换消去非最小耦合，得到的势在合适的耦合取值下能够支持暴涨。该模型颇具吸引力，因为它不依赖任何超出标准模型的新物理场，仅含一个自由参数，即该非最小耦合。文献 [194] 发现，渐近安全中可以预言非最小耦合的取值，至少在 G 和 Λ 满足汤川耦合可以非零的范围内确实如此。结果显示，预言得到的希格斯四次耦合与非最小耦合的比值 λ_4/ξ^2 过大，无法与宇宙微波背景数据兼容。如果这一结果在扩展截断中得到验证，就会排除渐近安全引力框架下的希格斯暴涨。该结论早在文献 [189] 中就通过对大场取值下希格斯势的研究得出了。

Physics Beyond the Standard Model

超出标准模型的物理学

There are indications that a complete description of nature requires particle physics beyond the SM. On the one hand, observations of galactic rotation curves, the cosmic microwave background, and gravitational lensing show that we do not fully understand the sources of gravity in the universe. A modification of the gravitational law itself remains a potential explanation, but missing matter is widely considered a more likely explanation. Primordial black holes may be one candidate but are only produced in sufficient abundance under appropriate conditions in the early universe. Note that it remains an interesting challenge to explore whether or not these are realizable within the asymptotic-safety paradigm. Alternatively, elementary particles beyond the SM are a candidate. These broadly fall into two categories, namely, weakly interacting massive particles (WIMPs), with masses roughly in the GeV range and couplings of the order of the weak interaction, and light particles with very small interaction strengths, such as axions or axion-like particles (ALPs). Both options have been investigated within the asymptotic-safety paradigm, including their interaction with gravity, and we will review the status in the following section.

有迹象表明，对自然的完整描述需要超出标准模型 (SM) 的粒子物理学。一方面，对星系旋转曲线、宇宙微波背景和引力透镜的观测表明，我们尚未完全理解宇宙中引力的来源。修改引力定律本身仍是一种潜在解释，但缺失物质被普遍认为是更可能的解释。原初黑洞可能是候选之一，但仅能在早期宇宙的合适条件下产生足够的数量。值得注意的是，探究这类候选能否在渐近安全范式中实现仍然是一个有趣的挑战。另一种观点则认为，超出 SM 的基本粒子就是暗物质候选。这类粒子大致分为两类: 一类是弱相互作用大质量粒子 (WIMP)，质量大致处于 GeV 量级，耦合强度约为弱相互作用的量级；另一类是相互作用强度极小的轻粒子，比如轴子或类轴子粒子 (ALP)。这两类可能性都已经在渐近安全范式中被研究，包括它们与引力的相互作用，我们将在下一节回顾研究现状。

On the other hand, there are also indications that the SM is not a complete description of visible matter. These indications come, for example, from the observation of neutrino oscillations, which prove that the neutrinos of the SM are massive, or from a tension between the measured and predicted value of the anomalous magnetic dipole moment of the muon. Adding new physics at the electroweak scale or above might solve these shortcomings of the SM. Within asymptotically safe quantum gravity, the gravitational impact on the anomalous magnetic dipole moment of the muon, the generation of neutrino masses, and grand unified theories have been studied explicitly. We will review their status in the following section.

另一方面，也有迹象表明 SM 并非对可见物质的完整描述。这些迹象例如来自中微子振荡的观测——它证明了 SM 中的中微子是有质量的，还来自缪子反常磁偶极矩的测量值与理论预测值之间的张力。在电弱标度或更高标度引入新物理或许能解决 SM 的这些不足。在渐近安全量子引力中，引力对缪子反常磁偶极矩的影响、中微子质量的产生以及大统一理论都已经得到了明确研究。我们将在下一节回顾它们的研究现状。

Higgs Portal to Dark Sectors

希格斯栅与暗 Sector

Synopsis: The portal coupling between the SM Higgs and a dark scalar, which is a popular, but increasingly tightly constrained coupling between the SM and a WIMP, is predicted to vanish in asymptotic safety. In contrast, a portal coupling to a dark sector with additional fields beyond the dark scalar may be generated either above or below the Planck scale. In contrast to phenomenological models of dark matter, such asymptotically safe portal models have a high predictive power.

概要: 标准模型希格斯玻色子与暗标量之间的栅耦合是标准模型与弱相互作用大质量粒子之间一种流行但约束日趋严格的耦合，渐进安全理论预言该耦合为零。与之不同的是，若暗 Sector 除暗标量外还存在额外场，栅耦合可在普朗克能标之上或之下生成。和暗物质唯象模型相比，这类渐进安全栅模型具备极高的预言能力。

A (massive) dark scalar d may couple to the Higgs scalar H of the SM through a portal coupling:

一个 (有质量) 暗标量 d 可通过栅耦合与标准模型的希格斯标量 H 耦合:

$$S = \lambda_p \int d^4x H^\dagger H d^2. \tag{37}$$

If the coupling is comparable to SM couplings, the dark scalar is in thermal equilibrium and thus produced through a standard freeze-out mechanism in the early universe. Whether or not a single scalar is a viable dark-matter candidate therefore depends on the size of the coupling. The coupling is also key to observational constraints, e.g., through the non-observation of scattering off SM particles in dedicated dark-matter experiments and non-observation of production at the LHC.

若该耦合的大小与标准模型耦合相当，暗标量会处于热平衡，因此在早期宇宙通过标准退耦机制生成。单个标量能否成为合格的暗物质候选者，完全取决于该耦合的大小。该耦合也是观测约束的核心，例如暗物质专门实验未观测到暗物质与标准模型粒子散射，大型强子对撞机也未观测到暗标量产生过程。

Similar to the quartic Higgs self-interaction, the gravitational contribution to λ_p is toward irrelevance at the free fixed point, which therefore is IR-attractive, and no other fixed point is generated:

和希格斯四次自相互作用类似，引力对 λ_p 的贡献使其在自由不动点处变为不相关算符，因此该不动点是红外吸引的，且不会生成其他不动点:

$$\beta_{\lambda_p}|_{\text{grav}} = -f_s \lambda_p \tag{38}$$

Furthermore, since no contributions from gauge or Yukawa couplings contribute to the beta function of λ_p , it is not regenerated below the Planck scale. Thus, $\lambda_p = 0$ is a prediction from asymptotic safety that holds at all scales [126].

此外，由于规范耦合和汤川耦合都不会对 λ_p 的 β 函数产生贡献，它不会在普朗克能标以下重新生成。因此，$\lambda_p = 0$ 是渐进安全得出的预言，在所有能标下都成立 [126]。

Intriguingly, experiments continuously improve the strength of bounds on λ_p but have not led to a detection, making the prediction from asymptotic safety compatible with the current experimental situation.

有趣的是，实验对 λ_p 的约束强度不断提升，但始终没有探测到信号，这使得渐进安全的预言与当前实验情况相容。

This result may be circumvented in a more complex dark sector which contains more than one field. For instance, the portal coupling is regenerated below the Planck scale, if a new gauge field couples to the Higgs and the dark scalar. Gauge field fluctuations generate the portal coupling below the Planck scale, leading to a prediction for the portal coupling as a function of the new gauge coupling [195,196]. As a second example, an additional dark fermion with a Yukawa coupling to the dark scalar can also generate the Higgs portal coupling [194, 195]. Dark fermions may even generate a portal coupling at transplanckian scales, i.e., in the fixed-point regime: in a two-step mechanism, a fixed point with a finite dark and visible Yukawa coupling necessarily induces a dark and visible nonminimal coupling. Together, the two nonminimal couplings generate a portal coupling. This model, although a toy model with an incomplete SM sector, is a striking example of the predictive power asymptotic safety may have: when viewed as an effective, phenomenological model, it has nine free parameters (two scalar masses, two quartic scalar couplings, one portal coupling, two nonminimal couplings, two Yukawa couplings). At an asymptotically safe fixed point, only the two mass parameters remain as free parameters. This reduces the parameter space of the model dramatically; see [106, 194].

如果是更复杂、包含不止一个场的暗区，这个结论可以规避。举例来说，如果一个新规范场同时耦合希格斯和暗标量，栅耦合就会在普朗克能标以下重新生成。规范场涨落会在普朗克能标以下生成栅耦合，得到栅耦合作为新规范耦合函数的预言 [195,196]。再比如，额外的暗费米子通过汤川耦合与暗标量相互作用，同样可以生成希格斯栅耦合 [194, 195]。暗费米子甚至可以在跨普朗克能标，也就是不动点区域生成栅耦合: 在两步机制中，暗和可见部分汤川耦合都有限的不动点，必然会诱导出暗和可见部分的非最小耦合。两个非最小耦合共同作用生成栅耦合。该模型虽然是只包含不完整标准模型 sector 的玩具模型，却十分有力地展示了渐进安全的预言能力: 如果将其视为有效唯象模型，它共有 9 个自由参数 (两个标量质量、两个标量四次耦合、一个栅耦合、两个非最小耦合、两个汤川耦合)。在渐进安全不动点处，仅剩下两个质量参数是自由参数，模型的参数空间被大幅缩减，参见 [106, 194]。

Similarly, [197] derives specific predictions for the masses of dark-matter particles, if the conformal Standard Model [198] becomes asymptotically safe. A similar predictive power was observed in a study [199] of BSM physics which provide a dark-matter candidate simultaneously with explaining the value of the muon $g-2$ measurement. There, many phenomenological models were ruled out when the coupling to gravity was included in a parameterized way, by including appropriate terms $\sim f_g, f_y$ into the beta functions; see also section ”Toward a UV Completion of the Standard Model”.

类似地，若共形标准模型 [198] 满足渐进安全，文献 [197] 推导出了暗物质粒子质量的具体预言。另一项研究 [199] 对超出标准模型的新物理得到了相同的预言能力，这类新物理既能给出暗物质候选者，同时可以解释缪子 $g-2$ 的测量结果。在该研究中，当通过在 β 函数中引入合适项 $\sim f_g, f_y$ 参数化地计入引力耦合后，许多唯象模型都被排除，另参见“走向标准模型的紫外完备”一节。

Further Reading

扩展阅读

Higgs Portal Couplings and Gauged B - L Symmetry

希格斯门户耦合与定域 B-L 对称性

A Higgs portal to a dark sector could also become relevant for models involving a gauged $B-L$ symmetry and thus a new gauge boson beyond the SM. In this case, the dark scalar field spontaneously breaks the $B-L$ symmetry. Imposing that these models become asymptotically safe under the inclusion of quantum gravity restricts the parameter space of new physics significantly. In particular, the kinetic mixing between the new gauge boson and the SM gauge bosons is fixed, and in some cases, the branching fraction of the new gauge boson to SM particles can be predicted; see [200]. This model can also be extended to accommodate fermionic dark matter, which is consistent with the observed relic density [201]. Imposing asymptotic safety allows constraining the mass of the dark-matter particles as a function of the mass of the new gauge boson.

希格斯到暗区的门户也可应用于包含定域 $B-L$ 对称性、因而超出标准模型存在一个新规范玻色子的模型。在这种情况下，暗标量场会自发破缺 $B-L$ 对称性。要求这些模型在引入量子引力后仍满足渐近安全，会对新物理的参数空间产生严格限制。具体而言，新规范玻色子与标准模型规范玻色子之间的动力学混合是确定的，部分情况下还可以预测新规范玻色子衰变到标准模型粒子的分支比，见文献 [200]。该模型还可以扩展为包含费米型暗物质的形式，与观测到的遗迹密度相符 [201]。要求满足渐近安全，可以根据新规范玻色子的质量对暗物质粒子的质量给出限制。

Axion-Like Particles in the Asymptotically Safe Landscape

渐近安全图景中的类轴子粒子

Synopsis: ALPs couple to the electromagnetic field through a dimension-five operator, which allows a conversion between photons and ALPs that enables experimental searches for ALPs. Within asymptotically safe gravity, there are indications that the ALP-photon coupling is driven to zero, unless gravity is strongly coupled. At strong coupling, there may be a tension with the weak-gravity bound (if it indeed exists), such that there may be a prediction from asymptotic safety, that the ALP-photon coupling vanishes.

概要: 类轴子粒子 (ALP) 通过五维算符与电磁场耦合，这使得光子和 ALP 之间可以发生相互转化，从而支持对 ALP 的实验搜寻。在渐近安全引力中，有迹象表明除非引力处于强耦合状态，否则 ALP 与光子的耦合会被驱动至零。在强耦合下，可能与弱引力边界 (如果该边界确实存在) 产生矛盾，因此渐近安全框架可能得出 ALP-光子耦合为零的预言。

The axion is a conjectured BSM particle which solves the strong CP problem. That ”problem” consists in the observation that the coupling of the term $F_{\mu\nu}\widetilde{F}^{\mu\nu}$ is very small or potentially zero in QCD. Note that for an Abelian gauge theory, $F_{\mu\nu}\widetilde{F}^{\mu\nu}$ is a total derivative, but not for a non-Abelian one. Furthermore, here, we have put quotation marks to highlight that the CP problem is not actually a consistency problem but a fine-tuning problem. Such fine-tuning problems start from the assumption that it is ”natural” for dimensionless numbers to be close to one. Therefore, a small number is said to require an explanation. We disagree with the expectation that a small number requires an explanation more than a number of order one does. Ultimately, a theory in which all free parameters become calculable from first principles would be most satisfying. In the absence of such a theory, any value can be chosen for a free parameter, and a particular deviation from 1 does not require less explanation than a particular deviation from 0 .

轴子是一种解决强 CP 问题的假想超出标准模型粒子。这个“问题”源于观测显示量子色动力学中 $F_{\mu\nu}\tilde{F}^{\mu\nu}$ 项的耦合非常小，甚至可能为零。请注意，对于阿贝尔规范理论，$F_{\mu\nu}\tilde{F}^{\mu\nu}$ 是全导数，但非阿贝尔规范理论并非如此。此外，我们在此处加上引号是为了强调，CP 问题实际上并非理论自洽性问题，而是精细调节问题。这类精细调节问题的出发点是如下假设: 无量纲量“自然”应该接近 1，因此小量需要额外解释。我们不认同“小量比一阶量更需要解释”的观点。归根结底，一个所有自由参数都可以从第一性原理计算得到的理论才是最令人满意的。如果不存在这样的理论，自由参数可以取任何值，偏离 1 并不比偏离 0 更不需要解释。

The strong CP problem can be solved by introducing an additional degree of freedom, namely, the axion, [202], which takes the place of the coupling of that term. A dynamical mechanism [203] drives the expectation value of the axion, i.e., the coupling of that term, to zero. Note that it remains an intriguing open question whether the work in [203] can be extended to a gravitational setting, where gravitational contributions to the anomalous dimension of the gauge field generate a flow for the coupling. This may solve the strong CP ”problem” without the need for new degrees of freedom. At the same time, a coupling between the axion field a and the electromagnetic field strength is generated, which takes the form:

强 CP 问题可以通过引入额外自由度解决，也就是轴子 [202]，由轴子替代原来的项耦合。动力学机制 [203] 将轴子的期望值 (也就是对应项的耦合) 驱动到零。值得注意的一个悬而未决的有趣问题是，文献 [203] 的结论能否推广到引力场景: 在引力场景中，引力对规范场反常维度的贡献会产生耦合的能动量流，这可能不需要引入新自由度就能解决强 CP“问题”。同时，轴子场 a 与电磁场强度会产生耦合，形式如下:

$$S_{\text{axion-photon}} = \int d^4x \bar{g}_a a F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{39}$$

ALPs are pseudoscalars, like the axion, and have the same coupling to photons as the axion. Axions and ALPs are very weakly coupled dark-matter candidates which can be generated out of equilibrium in the early universe; see [204] for a review.

ALP 和轴子一样是赝标量，与光子的耦合和轴子完全相同。轴子和 ALP 都是极弱耦合的暗物质候选者，可以在早期宇宙中偏离平衡产生；综述参见 [204]。

In string theory, the axion and ALPs are expected to exists [205]. If, therefore, they do not exist (or do not couple to photons) in asymptotic safety, that would be a discriminator between the two candidates for quantum gravity. In view of numerous searches for axion-photon and ALP-photon couplings, this discriminator is highly relevant.

在弦论中，轴子和 ALP 被认为存在 [205]。因此，如果在渐近安全框架中它们不存在 (或不与光子耦合)，这就会成为两种量子引力候选理论之间的区分特征。鉴于目前已有大量针对轴子-光子耦合与 ALP-光子耦合的搜寻实验，这一区分特征具有很高的现实意义。

In fact, the gravitational contribution to the flow of the ALP-photon coupling $g_a = \bar{g}_a k$ is toward relevance at the fixed point $g_{a*} = 0$. However, the gravitational contribution needs to overwhelm the canonical scaling dimension. Otherwise, the fixed point at $g_{a*} = 0$ cannot be connected to a nonzero ALP-photon coupling in the IR. The gravitational contribution can overwhelm the canonical scaling dimension, if gravitational fluctuations are strong enough. This is in conflict with the weak-gravity bound, if the latter indeed exists. In

[206], it is therefore concluded that the ALP-photon coupling may be predicted to vanish in asymptotically safe gravity.

实际上，引力对 ALP-光子耦合 $g_a = \bar{g}_a k$ 能动量流的贡献，会使其在不动点 $g_{a*} = 0$ 处向关联方向演化。但引力贡献需要压倒正则标度维数，否则 $g_{a*} = 0$ 处的不动点无法和红外区非零的 ALP-光子耦合连通。只有当引力涨落足够强时，引力贡献才能压倒正则标度维数。如果弱引力边界确实存在，这就会与它冲突。因此文献 [206] 得出结论: 在渐近安全引力中，ALP-光子耦合可以被预言为零。

Should the weak-gravity bound not persist, the ALP-photon coupling satisfies an upper bound in asymptotic safety, because schematically, the beta function is of the form:

如果弱引力边界不成立，那么在渐近安全框架中 ALP-光子耦合满足上界，因为概括来说 beta 函数形式为:

$$\beta_{g_a} = g_a + \beta_1 g_a G_N + \beta_2 g_a^3, \tag{40}$$

where β_1 is a function of the cosmological constant that is typically negative and β_2 is positive. The associated fixed-point structure is therefore the same as for the Abelian gauge coupling or some of the Yukawa couplings. The fixed point at $g_{a*} > 0$ hence imposes an upper bound on IR values of the ALP-photon coupling. In contrast to the Abelian gauge coupling and some of the Yukawa couplings, this fixed point only becomes available if $\beta_1 G_N < -1$, because of the canonical dimension of g_a .

其中 β_1 是宇宙常数的函数，通常为负，β_2 为正。因此对应的不动点结构和阿贝尔规范耦合或部分汤川耦合的不动点结构相同。因此 $g_{a*} > 0$ 处的不动点为 ALP-光子耦合的红外值给出了一个上界。和阿贝尔规范耦合以及部分汤川耦合不同，由于 g_a 的正则维数，只有当 $\beta_1 G_N < -1$ 时这个不动点才存在。

In summary, if the weak-gravity bound persists, ALP-photon couplings are likely driven to zero in asymptotic safety, implying that experiments will continue to place tighter constraints without a discovery. If the weak-gravity bound turns out to be a truncation artifact, an upper bound on the ALP-photon coupling exists. Both scenarios are experimentally testable and are in contrast to string theory.

总而言之，若弱引力边界成立，轴子样粒子与光子的耦合在渐近安全框架中会被趋近于零，这意味着实验只会不断收紧约束，而无法发现该粒子。若弱引力边界只是截断带来的人为结果，则轴子样粒子与光子的耦合仍存在上界。两种情况都可通过实验检验，且均与弦理论结论不同。

Grand Unified Theories

大统一理论

Synopsis: If the matter content of a grand unified theory is compatible with an asymptotically safe fixed point in gravity, then asymptotically safe GUTs are much more predictive than GUTs without gravity. The scalar potential for the many scalars that are required to spontaneously break the large gauge group to the SM gauge groups is completely fixed, except for a single parameter for each scalar field. Thereby, many breaking chains that are considered in gravity-free GUTs are no longer available in asymptotically safe GUTs.

概要: 如果大统一理论的物质内容与引力的渐近安全固定点相容，那么渐近安全大统一理论比不含引力的大统一理论预测能力强得多: 将大规范群自发破缺到标准模型规范群所需的多个标量场，其标量势除了每个标量场各对应一个自由参数外，完全固定。因此，不含引力的大统一理论中讨论的许多破缺链在渐近安全大统一理论中不再成立。

GUTs are attractive, because they explain charge quantization and explain why the charge of proton and electron are exactly equal in absolute value. Further, they can, upon spontaneous symmetry breaking to the SM gauge group, automatically give rise to a right-handed neutrino with the required SM charges. Besides this motivation, the near-crossing of the values of SM gauge couplings at energy scales of about 10^{16}GeV may be interpreted as an indication for a unification of the SM gauge groups to one larger group.

大统一理论颇具吸引力，因为它可以解释电荷量子化，解释为什么质子和电子的电荷绝对值恰好相等。此外，在自发对称性破缺到标准模型规范群后，大统一理论可以自动产生一个带有标准模型所需电荷的右手中微子。除了这些动机之外，标准模型规范耦合常数在约 10^{16}GeV 能标附近近似交汇，这一点可以被解读为标准模型规范群统一为更大群的证据。

However, GUTs are unattractive, because they come with a plethora of free parameters. These are linked to the scalar potential: in order to break the GUT gauge group to the SM gauge groups, several scalars are needed, which typically introduce numerous quartic couplings. There are multiple quartic invariants, because there are typically several scalar fields transforming in different representations of the gauge group. Depending on the representation, several different quartic invariants exist. In addition, quartic interactions can be built from quadratic interactions of two different scalars. In a typical GUT setting, these are free parameters. In turn, starting from a grand unified symmetry, many different chains of spontaneous symmetry breaking are available, depending on the values one chooses for these free parameters. It is therefore unexplained, why the SM, instead of a theory with a different symmetry, should come out as the low-energy limit of the GUT.

但大统一理论也存在缺陷，因为它含有大量自由参数。这些自由参数都与标量势相关: 为了将大统一理论的规范群破缺为标准模型规范群，需要多个标量场，这些标量场通常会引入大量四次耦合。由于一般存在多个标量场，分别对应规范群的不同表示，因此存在多个四次不变量。根据表示的不同，还会存在更多不同的四次不变量。此外，四次相互作用还可以由两个不同标量的二次相互作用构造而来。在典型的大统一理论框架中，这些都是自由参数。因此，从大统一对称性出发，选择不同的自由参数就可以得到多种不同的自发对称性破缺链。因此，大统一理论无法解释为什么低能极限是标准模型，而非其他不同对称性的理论。

In [207], it was proposed that if a GUT can become asymptotically safe under the impact of quantum gravity, then scalar potentials may largely be fixed. It is not conclusively established whether a GUT can become asymptotically safe under the impact of quantum gravity; the numerous matter fields of a GUT may even destroy the fixed point in the gravitational sector. Different studies come to different conclusions on this point; see, e.g., [80, 102]. Because these studies differ mainly in their choice of regulator function, the differing results may be interpreted as indicating the need to include further interactions in the studies. Specifically in [207], by the same mechanism as in section ”Higgs Quartic Coupling in the Standard Model” for the Higgs sector of the SM, all quartic couplings of the scalar potential may be predicted. The quadratic couplings, linked to the mass parameters, are expected to remain free parameters, but these add only a single free parameter for each scalar. In addition, the unified gauge coupling may also be predicted, by the same mechanism as the Abelian gauge coupling in the SM, if the matter content of the GUT is such that the gauge coupling is no longer asymptotically free [171].

文献 [207] 提出，如果大统一理论在量子引力的影响下可以变成渐近安全的，那么标量势可以在很大程度上被固定。目前还没有最终确定大统一理论能否在量子引力影响下成为渐近安全的；大统一理论中大量物质场甚至可能破坏引力区的固定点。不同研究对此得出了不同结论，参见例如 [80, 102]。由于这些研究的分歧主要来自规范函数的选择，不同的结果可能说明研究中需要纳入更多相互作用。特别是在文献 [207] 中，借助与“标准模型中的希格斯四次耦合”一节中标准模型希格斯区相同的机制，可以预测标量势的所有四次耦合。与质量参数相关的二次耦合预计仍为自由参数，但这也仅为每个标量增加一个自由参数。此外，如果大统一理论的物质内容使得规范耦合不再渐近自由 [171]，那么也可以借助与标准模型中阿贝尔规范耦合相同的机制预测统一规范耦合。

In turn, fixing the values of the quartic couplings removes much of the freedom in choosing scalar potentials to accommodate different breaking chains. As a consequence, one may expect that multiple breaking chains may be excluded and the asymptotically safe GUT setting may combine the attractive attributes of a GUT with the high predictive power of asymptotic safety, achieving explanations of many of the properties of the SM.

固定四次耦合的取值后，选择标量势适配不同破缺链的自由度会大大减少。因此，可以预期多重破缺链会被排除，渐近安全大统一框架可以结合大统一理论的优势与渐近安全的高预测能力，最终解释标准模型的诸多性质。

This general idea was put to the test in [208], where it was indeed shown that for SO(10) GUTs with a 16- and 45-dimensional scalar representation, particular breaking chains can be excluded. Such a result motivates further studies to determine whether (i) asymptotic safety can be achieved in gravity-GUT-theories and (ii) whether breaking chains to the SM are available or not.

这一总体思想已在文献 [208] 中得到检验: 该研究确实证明，对于包含 16 维和 45 维标量表示的 SO(10) 大统一理论，特定的破缺链可以被排除。这一结果推动进一步研究来确定:(i) 引力-大统一理论能否实现渐近安全；(ii) 破缺到标准模型的破缺链是否存在。

Neutrino Masses

中微子质量

Synopsis: Neutrino masses may be generated by adding a right-handed Weyl fermion to each generation in the SM, together with a Yukawa coupling to the Higgs field. At an asymptotically safe fixed point, such Yukawa couplings are automatically driven to zero. A crossover trajectory may therefore exist, which spends many scales close to such a fixed point and thereby drives the neutrino Yukawa coupling to tiny values, thus providing an explanation for the smallness of neutrino masses.

提要: 在标准模型中，可以为每代费米子添加一个右手外尔费米子，结合其与希格斯场的汤川耦合，即可产生中微子质量。在渐近安全不动点处，这类汤川耦合会被自动驱动至零。因此可能存在一条交叉轨迹，它在多个能标上都贴近该不动点，从而将中微子汤川耦合驱动到极小值，为中微子质量很小这一观测事实提供解释。

Alternatively, the seesaw mechanism may provide naturally small neutrino masses through heavy right-handed neutrinos. Asymptotic safety may constrain models based on the seesaw mechanism.

另一种机制是跷跷板机制，它可以通过重右手中微子自然获得小中微子质量。渐近安全可以对基于跷跷板机制的模型给出约束。

There are several possible ways to generate neutrino masses, two of which we will discuss, namely, the inclusion of right-handed Weyl neutrinos and the addition of Majorana masses for right-handed neutrinos.

产生中微子质量有多种可能的方案，我们将讨论其中两种: 引入右手外尔中微子，以及为右手中微子添加马约拉纳质量。

First, by adding a right-handed Weyl fermion to each generation of the SM, one can introduce a Yukawa coupling y_υ for neutrinos. To generate neutrino masses in the meV range, this Yukawa coupling has to be as small as $y_\upsilon \sim 10^{-13}$. In [182, 209, 210], it was shown that asymptotic safety may provide an explanation for such a small value: extending the studies of the Yukawa sector of the SM in [109, 170] (see section "Yukawa Couplings in the Standard Model"), by the neutrino Yukawa coupling, one finds two possible fixed points: one, at which all Yukawa couplings are asymptotically free, and a second one, at which top and bottom Yukawa coupling are nonzero. The first fixed point can accommodate the tiny IR value of the neutrino Yukawa coupling by choosing a suitable trajectory but cannot explain it. The second fixed point, where top and bottom Yukawa couplings are nonvanishing in the UV, predicts a vanishing neutrino Yukawa coupling and is therefore phenomenologically not viable. In combination, however, these two fixed points dynamically generate a tiny neutrino Yukawa coupling: starting from the asymptotically free fixed point, all Yukawa couplings grow, until the top and bottom Yukawa coupling reach the vicinity of the interacting fixed point. There, the critical exponent of the neutrino Yukawa coupling switches sign from positive (relevant) to negative (irrelevant), driving the neutrino Yukawa coupling back down to tiny values. Hence, the neutrino generically ends up much lighter than the other fermions.

首先，在标准模型的每一代中添加一个右手外尔费米子，就可以为中微子引入汤川耦合 y_υ。若要得到毫电子伏特量级的中微子质量，该汤川耦合必须小到 $y_\upsilon \sim 10^{-13}$ 。文献 [182, 209, 210] 指出，渐近安全可以解释这种极小的耦合值: 在标准模型汤川区研究 [109, 170](参见“标准模型中的汤川耦合”一节) 的基础上加入中微子汤川耦合后，可得到两种可能的不动点: 第一种是所有汤川耦合都渐近自由，第二种是顶夸克和底夸克的汤川耦合不为零。第一种不动点可以通过选择合适的轨迹容纳中微子汤川耦合的极小红外值，但无法解释它为何这么小。第二种不动点中顶夸克和底夸克汤川耦合在紫外区非零，但预测中微子汤川耦合为零，因此唯象上不成立。但这两种不动点结合可以动力学地产生极小的中微子汤川耦合: 从渐近自由不动点出发，所有汤川耦合都会增长，直到顶夸克和底夸克汤川耦合到达相互作用不动点附近。此时中微子汤川耦合的临界指数符号从正 (相关) 变为负 (无关)，将中微子汤川耦合重新压低到极小值。因此，中微子的质量 generically 会远小于其他费米子。

Because, without an asymptotically safe fixed point of the above type, no mechanism appears to exist to explain a tiny neutrino Yukawa coupling, this form of neutrino mass generation was long considered unappealing, because it is not ”natural.” Note that there is clearly some arbitrariness in the notion of naturalness: the ratio between electron mass and top-quark mass is already 10^{-6} but is usually not viewed as a motivation to think about alternative mass generation mechanisms for the electron. Instead, the line between ”natural” and ”unnatural” is in this case drawn somewhere below 10^{-6} , so that the ratio of about 10^{-9} between an meV-neutrino mass scale and the electron mass scale is considered ”unnatural.” This arbitrariness already suggests that naturalness may at best be a motivation to search for alternative explanations, but not a strong and unequivocal reason to rule an ”unnatural” setting out.

由于如果不存在上述类型的渐近安全不动点，就没有机制能解释极小的中微子汤川耦合，这种中微子质量产生形式长期以来都被认为缺乏吸引力，因为它不“自然”。注意，自然性的概念本身显然存在一定任意性: 电子质量和顶夸克质量的比值已经是 10^{-6}，但通常不会被认为是需要为电子寻找替代质量产生机制的动机。相反，在这个案例里，“自然”和“不自然”的界限被划在 10^{-6} 以下，因此毫电子伏特量级的中微子质量标度和电子质量标度之间约为 10^{-9} 的比值被认为是“不自然”的。这种任意性已经说明，自然性最多只能作为寻找替代解释的动机，而不能作为排除“不自然”框架的明确有力依据。

As an alternative, one can introduce heavy (with masses around the GUT scale) right-handed neutrinos with a Majorana mass term. The neutrino mass matrix, upon diagonalization, produces neutrino masses which are inversely proportional to the heavy Majorana mass scale, making neutrinos ”naturally” light.

作为替代方案，可以引入带有马约拉纳质量项的重右手中微子 (质量约为大统一能标)。中微子质量矩阵对角化后得到的中微子质量与重马约拉纳质量标度成反比，使得中微子“自然地”变得很轻。

Majorana masses were investigated in the context of asymptotically safe systems in [211], where they were found to remain relevant in presence of quantum gravity fluctuations. Accordingly, the corresponding mass scale can be chosen freely, providing a basis for the seesaw mechanism with heavy right-handed neutrinos. In [212], the seesaw mechanism for a specific choice of heavy fields was investigated and constrained. In particular, the additional fields have an impact on the prediction of the Higgs mass and top-quark mass, because both depend on physics at the new, heavy scale. Domènech et al. [212] therefore also constitutes an example for how an embedding into an asymptotically safe UV completion constrains not just the deep IR (around the electroweak scale) but also physics at intermediate scales which are beyond the reach of current experiments.

文献 [211] 在渐近安全系统的框架下研究了马约拉纳质量，发现在量子引力涨落存在时，马约拉纳质量仍然是相关算符。因此对应的质量标度可以自由选择，为带重右手中微子的跷跷板机制提供了基础。文献 [212] 对特定重场选择下的跷跷板机制进行了研究并给出了约束。具体来说，额外场会影响希格斯质量和顶夸克质量的预言，因为这两个质量都依赖于新重能标处的物理。因此 Domènech 等人的工作 [212] 也说明了，嵌入渐近安全紫外完备不仅会约束深红外区 (电弱能标附近)，也会约束当前实验无法触及的中间能标物理。

$g - 2$ and Flavor Anomalies

$g - 2$ 与味反常

Synopsis: There might be a possibility for new physics at the electroweak scale, in order to resolve tensions between SM predictions and experimental data on the muon magnetic moment and on lepton flavor nonuniversality in rare B meson decays. Among the phenomenological models that have been proposed, asymptotic safety could act as a discriminator, because its predictive power may rule out values of couplings which are required to resolve the tensions.

摘要: 为解决标准模型预测与 μ 子磁矩、稀有 B 介子衰变轻子味非普适性实验数据之间的矛盾，电弱标度可能存在新物理。在已提出的唯象模型中，渐近安全可作为筛选标准，其预测能力可以排除解决这些矛盾所需的部分耦合取值。

Current experimental data on parameters of the SM indicate several anomalies, i.e., discrepancies between measurement and theoretical prediction. Since these discrepancies are below a statistical significance of 5σ , they are not discoveries but merely anomalies. Future experiments and updated theoretical methods will either resolve the tension or increase its significance, possibly beyond 5σ . The most commonly discussed anomalies concern the anomalous magnetic moment of the muon, $(g-2)_\mu$, and flavor anomalies in the $b \rightarrow s$ and the $b \rightarrow c$ transitions. While these anomalies do not provide sufficient evidence for a significant deviation from SM predictions (yet), many models involving particle physics beyond the SM were developed to explain them.

当前标准模型参数的实验数据显示出多处反常，即测量与理论预测之间的偏差。由于这些偏差的统计显著性低于 5σ ，它们并非正式发现，仅属于反常现象。未来实验与更新的理论方法要么会解决这一矛盾，要么会提高其显著性，甚至可能超过 5σ 。目前讨论最多的反常包括 μ 子反常磁矩 $(g-2)_\mu$ ，以及 $b \rightarrow s$ 和 $b \rightarrow c$ 跃迁中的味反常。尽管 (目前) 这些反常尚未提供足够证据证明存在偏离标准模型预测的显著偏差，但人们已经建立了许多超出标准模型的粒子物理模型来解释它们。

Some of these models have been investigated in the context of asymptotically safe quantum gravity; see [199, 213, 214]. In particular, it was investigated whether quantum gravity might turn some parameters of the extensions into irrelevant directions. In this case, the predictive power of asymptotically safe quantum gravity would extend to physics beyond the SM and predict, for example, the mass of dark-matter particles that are required to resolve the anomaly. Confronting these predictions with existing bounds from searches for physics beyond the SM can either rule out such solutions or provide strong constraints on the parameter space. These constraints might guide experimental searches and allow insights in the most promising next-generation particle colliders. For instance, in [213], the leptoquark solution to flavor anomalies was investigated, and it was found that asymptotic safety limits the mass range of the leptoquark to $4-7\text{TeV}$, where it is within reach of future colliders.

其中部分模型已在渐近安全量子引力框架下开展研究；参见 [199, 213, 214]。具体而言，研究者已经探究了量子引力是否会将超出标准模型扩展的部分参数转变为无关方向。在这种情况下，渐近安全量子引力的预测能力可以延伸到超出标准模型的物理，例如预测解决反常所需的暗物质粒子质量。将这些预测与超出标准模型新物理搜寻得出的现有限制作对比，既可以排除这类解决方案，也可以对参数空间给出强约束。这些约束可以指导实验搜寻，帮助人们判断最具前景的下一代粒子对撞机方向。例如，文献 [213] 研究了用轻夸克解释味反常的方案，发现渐近安全将轻夸克的质量范围限制在 $4-7\text{TeV}$ ，该范围处于未来对撞机的探测能力之内。

On the technical level, these studies proceed within a parameterized framework, first introduced in [170], in which the gravity contributions are parameterized by f_c ’s and resulting fixed points in matter beta functions are investigated. These studies therefore provide experimentally testable consequences of asymptotic safety under the assumption that asymptotic safety is realized in the full system. Checking this assumption would require (i) to account for the impact of the new matter fields on the gravitational fixed point to check whether it exists, (ii) to calculate the resulting values of f_c to compare with those required on a phenomenological level, and (iii) to check that higher-order interactions in the matter sector as well as nonminimal interactions between matter and gravity are subleading and do not change the conclusions much.

在技术层面，这类研究在一个最早由文献 [170] 提出的参数化框架中开展，该框架用 f_c 参数化引力贡献，并研究物质 β 函数中由此产生的不动点。因此，这类研究在“渐近安全可在整个系统中实现”的假设下，给出了渐近安全可被实验检验的推论。验证该假设需要:(i) 考虑新物质场对引力不动点的影响，检验该不动点是否存在；(ii) 计算得出 f_c 的结果值，与唯象层面要求的取值进行对比；(iii) 检验物质区域的高阶相互作用以及物质与引力之间的非最小相互作用都是次领头阶的，不会大幅改变结论。

On the Near-Perturbative Nature of Gravity-Matter Systems

引力-物质系统的近微扰性质

Synopsis: An asymptotically safe fixed point can be near perturbative, i.e., be close to canonical power counting. For such a fixed point, calculations are easier and systematic uncertainties are simpler to control. The SM coupled to gravity may have such a near-perturbative fixed point which is easy to connect to the perturbative RG flow of the SM at and below the Planck scale. More generally, phenomenological studies of asymptotically safe gravity-matter systems typically rely on the assumption that the system is near perturbative.

概要: 渐近安全固定点可以是近微扰的，即接近正则幂计数。对于这类固定点，计算更为简便，系统不确定性也更易控制。耦合引力的标准模型可能拥有这类近微扰固定点，它很容易与普朗克能标及以下能标的标准模型微扰重整化群流联系起来。更一般地说，渐近安全引力-物质系统的唯象研究通常依赖于系统近微扰这一假设。

In the previous sections, we relied on an implicit assumption about the nature of the systems we investigated: this assumption is implicit in the beta functions we used, which are all limited to leading order terms (i.e., low orders in the couplings) and to the canonically least irrelevant interactions of the system.

在前文中, 我们依赖了一个关于所研究系统性质的隐含假设: 该假设隐含在我们使用的 β 函数中, 这些 β 函数全都限于领头阶项 (即耦合中的低阶)，并且限于系统正则最不相关相互作用。

This assumption is that the matter-gravity systems we investigated are sufficiently weakly coupled to be near perturbative, despite being asymptotically safe. Technically, this makes robust calculations possible. Physically, this goes hand in hand with the idea that to control the (trans)planckian regime in a quantum field theory of gravity, a mechanism of dynamical weakening has to apply to quantum gravity.

这个假设就是，我们研究的物质-引力系统尽管渐近安全，但耦合足够弱，处于近微扰范围。从技术层面来说，这让可靠计算成为可能。从物理层面来说，这与下述观点一致: 若要在量子引力场论中控制 (跨) 普朗克区，量子引力必须适用动力学弱化机制。

To provide the technical and conceptual underpinning of the discussion in the previous sections, we therefore define in more detail what we mean by near perturbative and discuss the evidence for the near-perturbative nature of gravity-matter systems.

因此，为了给前文的讨论提供技术与概念基础，我们更详细地定义近微扰的含义，并讨论支持引力-物质系统具有近微扰性质的证据。

In general, interacting systems can be strongly coupled and governed by non-perturbative effects or can be weakly coupled and governed by perturbative effects or anything in between. This has phenomenological implications - for instance, in strongly coupled systems, the fundamental degrees of freedom can bind together and form new, stable, or unstable states. It also has formal implications - most importantly for the set of tools that is best to analyze the system and also for the type of approximations that can be made.

一般而言，相互作用系统可以是强耦合、由非微扰效应主导，也可以是弱耦合、由微扰效应主导，或是介于两者之间。这会产生唯象学影响——例如，在强耦合系统中，基本自由度可以结合在一起，形成新的稳定或不稳定态。它还会产生形式层面的影响——最重要的是影响最适合分析系统的工具集合，以及可采用的近似类型。

For gravity in the UV, one may first expect that it is strongly coupled and non-perturbative. This expectation arises, because the dimensionless Newton coupling, G_{N} , grows when one goes from low to high momenta. If one simply extrapolates from the classical regime, where $G_{\mathrm{N}}(k) \sim k^2, G_{\mathrm{N}}$ becomes of order one at the Planck scale. This is typically interpreted as a sign of strong coupling. We caution that values of couplings are not a good measure of perturbativity, because, e.g., the fixed-point value of a coupling can be changed arbitrarily by rescalings of the coupling.

对于紫外引力，人们起初可能会认为它是强耦合且非微扰的。这一猜想源自: 无量纲牛顿耦合 G_{N} 会随着动量从低到高不断增大。如果直接从经典区域外推，在普朗克尺度下 $G_{\mathrm{N}}(k) \sim k^2, G_{\mathrm{N}}$ 会达到一阶的量级。这通常被解读为强耦合的标志。我们提醒，耦合常数的数值并不是衡量微扰性的好方法，例如，耦合的定点值可以通过重新缩放耦合任意改变。

However, several sets of results indicate that asymptotically safe gravity-matter systems are near perturbative at high energies. With near perturbative, we refer to a situation where the theory is interacting, i.e., not strictly perturbative, but at the same time lacks nonperturbative phenomena such as the formation of stable bound states. Such near-perturbative behavior of asymptotically safe gravity-matter systems is indicated by (i) the critical exponents of higher-order interactions which remain near-canonical, (ii) the contributions of quantum gravity to beta functions in the matter sector, which are small, and (iii) symmetry identities between gravity-matter interactions which are near trivial, as they are in perturbative settings. We will discuss each of these indications in the following.

但多组结果表明，渐近安全引力-物质系统在高能下是近微扰的。我们所说的近微扰，指的是理论是相互作用的，即非严格微扰，但同时不存在非微扰现象，比如稳定束缚态的形成。渐近安全引力-物质系统的这类近微扰行为由以下几点佐证:(i) 高阶相互作用的临界指数保持近正则，(ii) 量子引力对物质区 β 函数的贡献很小，(iii) 引力-物质相互作用之间的对称性恒等式近乎平凡，和微扰环境中的情况一致。我们接下来将逐一讨论这些佐证。

Canonically Irrelevant Couplings Remain Irrelevant

经典无关耦合保持无关性

At a noninteracting fixed point, the critical exponents of all couplings correspond to their canonical mass dimension. At an interacting fixed point, the critical exponents still contain a dimensional contribution, but also an additional contribution δ_i which is induced by quantum fluctuations, i.e.,

在非相互作用不动点处，所有耦合的临界指数都对应其经典质量维度。在相互作用不动点处，临界指数仍包含维度贡献，但还存在由量子涨落诱发的额外贡献 δ_i ，即

$$\Theta_i = d_{\bar{g}_i} + \delta_i. \tag{41}$$

If the quantum contributions δ_i grow very large, they can turn canonically irrelevant couplings into relevant directions at an interacting fixed point. This would decrease the predictivity of the system, compared to the non-interacting fixed point, since more relevant directions indicate more free parameters that need to be fixed by experiments. More importantly, this would indicate that the system is very nonperturbative, since quantum fluctuations drastically change qualitative features of the system. Conversely, an interacting fixed point where all canonically irrelevant couplings remain irrelevant with critical exponents close to their canonical mass dimension is near perturbative and quantum fluctuations only change quantitative features, such as the value of couplings at low energies.

如果量子贡献 δ_i 变得非常大，它们就能在相互作用不动点处将经典无关耦合转变为相关方向。与非相互作用不动点相比，这会降低系统的可预测性，因为更多相关方向意味着更多需要通过实验确定的自由参数。更重要的是，这表明系统具有强非微扰性，因为量子涨落彻底改变了系统的定性特征。反之，若所有经典无关耦合在相互作用不动点仍保持无关，且临界指数接近其经典质量维度，则该不动点是近微扰的，量子涨落仅改变定量特征，例如低能下的耦合取值。

In all studies so far, the critical exponents of interactions involving matter fields follow their canonical mass dimension. In particular, canonically irrelevant matter interactions [106, 110, 111, 126, 127, 130, 133] and nonminimal gravity-matter interactions [91,93,94,96,98,106] remain irrelevant at the asymptotically safe fixed point. This justifies truncations based on canonical power counting a posteriori.

迄今为止所有研究中, 包含物质场的相互作用的临界指数都符合其经典质量维度。具体而言, 经典无关物质相互作用 [106, 110, 111, 126, 127, 130, 133] 和非最小引力-物质相互作用 [91,93,94,96,98,106] 在渐近安全不动点仍保持无关。这为基于经典幂次计数的截断提供了事后验证。

The Impact of Gravity on the Matter Sector

引力对物质部分的影响

As mentioned previously, the strength with which gravitational fluctuations impact the matter sector is encoded in effective gravitational couplings:

如前所述，引力涨落对物质部分的作用强度被编码在有效引力耦合中:

$$G_{\text{eff}}^{(n)} = \frac{G_{\text{N}}}{(1-2\Lambda)^n}. \tag{42}$$

Accordingly, the impact of gravity on the matter sector becomes weaker, the more negative the fixed-point value of the cosmological constant, and the smaller the fixed-point value of the Newton coupling gets. Independent of the details of the setup, fermionic matter was found to decrease the effective gravitational coupling [91]. Similarly, gauge fields decrease $G_{\text{N, eff}\,n}$, since the fixed-point value for G_{N} approaches zero for $N_{\text{V}} \to \infty$. While scalar fields might increase $G_{\text{N, eff}\,n}$, the effective gravitational couplings for the field content of the SM remain small.

因此，宇宙学常数的不动点值越负，牛顿耦合的不动点值越小，引力对物质部分的作用就越弱。不依赖于装置的细节，研究已发现费米物质会降低有效引力耦合 [91]。类似地，规范场会降低 $G_{\text{N, eff}\,n}$ ，这是因为 G_{N} 的不动点值对 $N_{\text{V}} \to \infty$ 趋近于零。虽然标量场可能会增大 $G_{\text{N, eff}\,n}$ ，但标准模型场组分的有效引力耦合仍然很小。

As a consequence, the gravitational contribution to the scale dependence of matter couplings is subleading compared to matter contributions, such that the matter sector might remain near perturbative at the UV-fixed point. This can also be seen by explicitly evaluating f_g or f_y , e.g., at the fixed point found in [80], as was done in [167], finding $f_g = 0.048$. In particular, the observation of nonvanishing fermion masses requires a small-enough impact of gravity on the matter sector, as discussed in Fig. 5; see also [107].

因此，与物质贡献相比，引力对物质耦合标度依赖性的贡献是次要的，因此在紫外不动点处物质部分可以保持近微扰性。这也可以通过显式计算 f_g 或 f_y 得到验证，例如如文献 [167] 所做的，在文献 [80] 找到的不动点处进行计算，得到了 $f_g = 0.048$ 。特别地，非零费米子质量的观测要求引力对物质部分的作用足够小，如图 5 所讨论的；参见文献 [107]。

These results imply that the asymptotically safe fixed point might provide a straightforward UV completion of the SM, which is perturbative at the Planck scale. Conversely, one may interpret the fact that the SM couplings are perturbative at the Planck scale as an indication that a UV completion with gravity must be near perturbative.

这些结果表明，渐近安全不动点可以为普朗克尺度下的微扰标准模型提供一个直接的紫外完备化。反过来看，我们可以将标准模型耦合在普朗克尺度下是微扰的这一事实，解读为引力紫外完备化必须是近微扰的一个标志。

Nontrivial Symmetry Identities

非平凡对称恒等式

Just like in Abelian and non-Abelian gauge theories, one breaks the gauge symmetry of gravity when computing gravitational fluctuations with the FRG. As a consequence, the scale dependence of different gravity-matter vertices differs from one another. In other words, the scale dependence of the Newton coupling, when read-off from different vertices, differs. This however does not mean that diffeomorphism invariance is manifestly broken. Instead, nontrivial symmetry identities, the Slavnov-Taylor identities, encode how diffeomorphism invariance is restored. If these identities are solved together with the scale dependence of the effective action, diffeomorphism invariance is retained along the RG trajectory.

就像在阿贝尔与非阿贝尔规范理论中一样，使用泛函重整化群 (FRG) 计算引力涨落时会破缺引力的规范对称性。由此导致，不同引力-物质顶点的标度依赖性各不相同。换句话说，从不同顶点读取得到的牛顿耦合的标度依赖性存在差异。但这并不意味着微分同胚不变性被明显破缺。相反，非平凡对称恒等式即斯拉夫诺夫-泰勒恒等式，刻画了微分同胚不变性是如何恢复的。如果将这些恒等式与有效作用量的标度依赖性联立求解，微分同胚不变性在整个重整化群 (RG) 轨迹上都能得以保留。

Naively, these identities are trivial in the perturbative regime, where a single gauge coupling can be defined. In Yang-Mills theories, the one-loop universality of beta functions ensures exactly this property: the scale dependence for the gauge coupling is identical, when extracted from pure gauge or gauge-ghost vertices. In the nonperturbative regime however, the scale dependence of different vertices disagrees and can even have opposite signs; see [215].

简单来看，这些恒等式在微扰区域是平凡的，在该区域可以定义单一规范耦合。杨-米尔斯理论中，β 函数的单圈普适性恰好保证了这一性质: 无论是从纯规范顶点还是规范-鬼顶点提取得到的规范耦合，其标度依赖性都是一致的。但在非微扰区域，不同顶点的标度依赖性并不一致，甚至符号相反；参见文献 [215]。

In asymptotically safe gravity, different gravity-matter vertices are found to agree on a semiquantitative level at the fixed point [87, 89-91] but can disagree significantly away from the fixed point.

在渐近安全引力中，已发现不同引力-物质顶点在不动点处达到半定量一致 [87, 89-91]，但在远离不动点的区域一致性会显著失效。

The semiquantitative agreement is defined in [89, 90] by comparing the scale dependences of different gravity-matter vertices and setting all versions of the Newton coupling equal to one another. If these differences between scale dependences are zero, one unique Newton coupling can be defined. If these differences are large, all different vertices have to be treated independently to fully capture the UV-behavior of asymptotically safe gravity-matter systems. In analogy to QCD, a semiquantitative agreement between different vertices at the fixed-point indicates that the theory might be near perturbative. In particular, it might imply that the underlying Slavnov-Taylor identities are trivial at the fixed point.

半定量一致性由 [89, 90] 定义，具体做法是比较不同引力-物质顶点的标度依赖关系，并令所有版本的牛顿耦合彼此相等。若这些标度依赖之间的差异为零，就可以定义唯一的牛顿耦合。若这些差异很大，则必须独立处理所有不同顶点，才能完整捕捉渐近安全引力-物质系统的紫外行为。类比量子色动力学，不同顶点在不动点处的半定量一致性表明该理论可能接近微扰。特别地，这可能意味着基础的斯拉沃诺夫-泰勒恒等式在不动点处是平庸的。

This provides another piece of evidence that asymptotically safe quantum gravity might be near perturbative: it is nonperturbative enough to induce scale invariance at high energies but remains as perturbative as possible.

这为渐近安全量子引力可能是近微扰的提供了另一项证据: 它具备足够的非微扰性以在高能下诱导出标度不变性，但同时尽可能保留了微扰性质。

Summary, Outlook, and Open Questions

总结、展望与开放问题

What are the key challenges in high-energy physics today? The first key challenge is to test the quantum nature of gravity, i.e., predict observable and testable consequences from candidate quantum theories of gravity. In this field, predictions from fundamental theory are rare, and most results are based on phenomenological models. The second key challenge is to solve open problems in particle physics, such as the nature of dark matter, the origin of the free parameters of the Standard Model, the origin of matter-antimatter asymmetry, and the mechanism for neutrino mass generation. In this field, there is a vast collection of more or less ad hoc phenomenological models.

当今高能物理学的核心挑战是什么？第一项核心挑战是检验引力的量子属性，即从候选量子引力理论出发，得出可观测、可检验的推论。该领域中，基础理论的推论十分稀缺，多数研究结果都建立在唯象模型之上。第二项核心挑战是解决粒子物理学的开放问题，例如暗物质的本质、标准模型自由参数的起源、正反物质不对称的起源，以及中微子质量的产生机制。该领域中存在大量或多或少都属于特设的唯象模型。

In this section, we have summarized research that aims at addressing both challenges at once. The key assumption underlying this line of research posits that only by addressing both challenges at once can one find a meaningful solution: predictions from fundamental theory require a quantum theory of gravity and matter as their starting point; such a theory in turn is expected to have high predictive power and thus select among the ad hoc phenomenological models often proposed to solve specific problems in particle physics.

在本章节中，我们总结了旨在同时应对这两项挑战的研究。该研究方向的核心假设认为，只有同时应对两项挑战才能找到有意义的解: 基础理论的预言需要以量子引力与物质的量子理论作为出发点；反过来，这样的理论有望拥有极高的预言能力，从而可以为解决粒子物理特定问题而提出的各类特设唯象模型做出筛选。

The paradigm in which such a quantum theory of gravity and matter is being developed is the asymptotic-safety paradigm. It requires quantum scale symmetry at UV scales. In turn, the presence of this symmetry constrains both the UV and IR properties of the theory. Constraints on the IR, where the symmetry is no longer realized, arise because a departure from scale symmetry is only possible for the relevant parameters of the theory, which are very few. To use a literary analogy, quantum scale symmetry is like the Cheshire cat: even when it is no longer present, its smile remains behind - the smile being the relations between couplings in the theory that hold in the IR.

这类量子引力与物质理论是在渐近安全范式下发展而来的。该范式要求紫外能标存在量子标度对称性，这一对称性同时会约束理论的紫外与红外性质。在红外区域对称性不再显现，约束依然存在，因为只有极少数理论的相关参数能够偏离标度对称性。打一个形象的比方，量子标度对称性就像柴郡猫: 即便它消失了，笑容却留了下来——这份笑容就是理论中耦合之间在红外依然成立的关系。

General Summary

概述

The current state of the art suggests that asymptotic safety of gravity with matter may indeed be realized for Standard Model-like theories, i.e., theories with the degrees of freedom of the Standard Model, and possibly a few additional degrees of freedom and with coupling values close to or exactly those of the Standard Model. There is evidence that, under the impact of the matter fields of the Standard Model, gravity is asymptotically safe. In turn, under the impact of asymptotically safe gravity, the Standard Model couplings become asymptotically free; and the Higgs quartic coupling emerges as a calculable quantity in the IR. There is even evidence of higher predictive power, where the Abelian gauge coupling also emerges as a calculable quantity in the IR; and potentially some of the Yukawa couplings do as well. The status of these predictions is as follows: the existence of predictive (partial) fixed points for the Abelian gauge coupling and the Yukawa couplings is a robust result (for the Yukawa couplings, this constrains the gravitational fixed-point values); however, the resulting IR values of the couplings are only known within significant systematic uncertainties. Therefore, it is currently not known whether or not the predictions match observations or not.

目前的研究进展表明，带物质的引力渐近安全确实可以在类标准模型理论中实现，即这类理论拥有标准模型的自由度，可能仅新增少量自由度，且耦合值接近或完全等于标准模型的耦合值。已有证据显示，在标准模型物质场的作用下，引力是渐近安全的。反过来，在渐近安全引力的作用下，标准模型耦合变为渐近自由；希格斯四次耦合成为红外区域可计算的量。甚至有证据显示该理论具备更高的预言能力: 阿贝尔规范耦合也成为红外区域可计算的量，部分汤川耦合可能也满足这一点。这些预言的现状如下: 阿贝尔规范耦合和汤川耦合存在可预言的 (部分) 不动点是可靠结论 (对汤川耦合而言，这一点约束了引力不动点的值)；但耦合对应的红外值目前仍存在很大的系统不确定性。因此，目前尚无法确定这些预言是否与观测结果相符。

The current state of the art also includes theories beyond the Standard Model, which are significantly constrained by the predictive power of asymptotic safety. Among a growing number of theories that have been explored, most noteworthy is the high predictive power for theories including dark-matter candidates. Among popular proposals, such as a Higgs portal to a dark scalar, an axion-like-particle, or a dark photon, many possibilities are, according to the most advanced calculations in these settings, ruled out.

目前的研究进展也涵盖了超出标准模型的理论，这些理论被渐近安全的预言能力大幅约束。在已被探索的越来越多的理论中，最值得关注的是其对包含暗物质候选者的理论的高强预言能力。在希格斯门户暗标量、类轴子粒子、暗光子这类主流理论方案中，根据该方向目前最先进的计算结果，许多可能性都已被排除。

These successes of the asymptotic-safety paradigm for gravity with matter have by now given rise to a phenomenological approach to such theories, which one may call a principled-parameterizedapproach, first put forward in [170]: the approach consists in adding contributions to the beta functions of a matter theory. These contributions come with free parameters (thus ”parameterized” approach); but their dependence on the couplings of the theory and the scale at which they are important are that of quantum gravity (thus ”principled” approach). This approach is based on the observation that quantum-gravity contributions to matter couplings are ”blind” to internal symmetries; thus, for instance, the quantum-gravity contribution to the beta function of a gauge coupling does not depend on the gauge group. Therefore, in the principled-parameterized approach, one assumes that a fixed point is present in the gravity theory including the matter fields of the theory. One then parameterizes the effect of gravity on the matter couplings, searches for fixed points, and analyzes their predictive power. Due to its relative technical simplicity (compared to fully-fledged calculations of the gravitational contributions), this principled-parameterized approach holds significant promise to solve the two key challenges in high-energy physics: first, it allows to derive observable consequences of quantum gravity. Second, it allows to search for fundamental theories which solve outstanding problems in particle physics.

带物质引力渐近安全范式的这些成果，如今已经催生了针对这类理论的唯象学研究方法，可称之为原则参数化方法，由文献 [170] 首次提出: 该方法的核心是在物质理论的 β 函数中添加贡献项。这些贡献项带有自由参数 (因此叫“参数化”方法)；但它们对理论耦合的依赖关系，以及它们发挥作用能标，都符合量子引力的特征 (因此叫“原则”方法)。该方法基于以下观测结论: 量子引力对物质耦合的贡献对内部对称性是“不敏感”的；因此举例来说，量子引力对规范耦合 β 函数的贡献不依赖于规范群。因此，在原则参数化方法中，人们假设包含理论物质场的引力理论中存在一个不动点，之后对引力对物质耦合的效应做参数化处理，搜索不动点并分析它们的预言能力。由于和完整计算引力贡献相比，该方法在技术上相对简单，因此原则参数化方法对解决高能物理的两大核心问题极具前景: 第一，它可以推导出量子引力的可观测效应；第二，它可以帮助寻找能解决粒子物理学悬而未决问题的基本理论。

Detailed Summary of Results

结果详细总结

In more detail, the results discussed above are based on the following discoveries. All of these arise within truncated renormalization group studies and are therefore subject to systematic uncertainties. However, the results quoted below have all been tested for their robustness, e.g., by extending the truncations considered. They may therefore be expected to hold, unless strongly nonperturbative effects in the path integral become relevant. As discussed in section ”On the Near-Perturbative Nature of Gravity-Matter Systems”, there is compelling evidence that assuming a near-perturbative gravity-matter system is self-consistent for the Standard Model and Standard-Model like systems.

更详细地说，上文讨论的结果基于以下发现。所有这些发现均来自截断化重整化群研究，因此存在系统不确定性。但下文列出的结果都已通过扩展截断范围等方式验证了稳健性。因此可以预期这些结果成立，除非路径积分中强烈的非微扰效应变得相关。正如“引力-物质系统的近微扰性质”一节所讨论的，有令人信服的证据表明，对于标准模型及类标准模型系统，假设近微扰引力-物质系统是自洽的。

- Quantum fluctuations of matter impact the gravitational fixed point but leave it at real fixed-point coordinates for the number of fields in the Standard Model. Therefore, the compelling evidence for asymptotic safety in gravity is the starting point to construct asymptotically safe gravity-matter models.

- 物质的量子涨落会影响引力不动点，但对于标准模型的场数，该不动点仍保持在实不动点坐标上。因此，引力渐近安全性的可靠证据是构建渐近安全引力-物质模型的出发点。

- Quantum fluctuations of gravity generate interaction terms for matter fields. This general result may be expected, because (i) gravity is interacting in the asymptotic-safety paradigm and (ii) matter cannot be decoupled from gravity. Therefore self-interactions of the different matter fields and interactions between them are expected and indeed found in explicit computations. As a nontrivial result, the generated interactions obey the maximum global symmetries of the kinetic terms of the matter fields. Generated interactions thus include, e.g., chirally symmetric four-fermion couplings and shift-symmetric scalar self-interactions.

- 引力的量子涨落会为物质场生成相互作用项。这一普遍结论符合预期，因为 (i) 在渐近安全范式中引力本身就是相互作用的，(ii) 物质无法与引力解耦。因此不同物质场的自相互作用以及它们之间的相互作用是预期存在的，并且确实在显式计算中被找到了。一个重要结论是，生成的相互作用满足物质场动能项的最大整体对称性。因此生成的相互作用包括例如手征对称四费米子耦合和平移对称标量自相互作用。

- Working toward the Standard Model of particle physics, several prerequisites need to be checked, for example, whether gravity, similarly to non-Abelian gauge interactions, can trigger chiral symmetry breaking and generate fermion masses. If it would do so, those masses should be close to the Planck scale, being inconsistent with models with light fermions, such as the Standard Model.

- 为构建粒子物理标准模型，需要验证多个前提条件，例如引力能否类似非阿贝尔规范相互作用触发手征对称性破缺并生成费米子质量。如果引力确实能做到这一点，这些质量就会接近普朗克尺度，这与标准模型这类包含轻费米子的模型矛盾。

Despite being an attractive interaction, the fluctuations of gravity do not trigger the formation of bound states and associated chiral symmetry breaking, allowing fermions to be light, with masses well below the Planck scale. However, spacetime curvature is bounded to avoid gravitational catalysis. In turn, because quantum fluctuations of matter impact gravitational fixed-point values and thus the solution to the effective equations of motion at high curvature, avoiding gravitational catalysis generates bounds on the number of matter fields that may exist in asymptotically safe gravity-matter systems. Therefore, gravitational fluctuations do not induce the breaking of chiral symmetry, such that models with light fermions, such as the Standard Model, can be compatible with asymptotically safe quantum gravity.

尽管引力是吸引力，但其涨落并不会触发束缚态形成以及相应的手征对称性破缺，这允许费米子保持为远低于普朗克尺度的轻质量。但要避免引力催化，时空曲率必须存在界限。反过来，由于物质的量子涨落会影响引力不动点的值，进而影响高曲率下有效运动方程的解，避免引力催化就对渐近安全引力-物质系统中可存在的物质场数目给出了限制。因此引力涨落不会诱发手征对称性破缺，因此标准模型这类包含轻费米子的模型可以与渐近安全量子引力相容。

- Gravity is blind to internal symmetries, such that the gravitational contribution to the running of all gauge couplings is the same, irrespective of the gauge group. Similarly, gravity contributes to all Yukawa couplings in the Standard Model in the same way, irrespective of whether one considers quarks or leptons and irrespective of the flavor. Finally, gravity also contributes to quartic scalar couplings with a universal contribution.

- 引力对内对称性无区分，因此无论规范群如何，引力对所有规范耦合跑动的贡献都是相同的。同理，引力对标准模型中所有汤川耦合的贡献也相同，与所考虑的是夸克还是轻子无关，也与味道无关。最后，引力对标量四次耦合也给出统一贡献。

- In the Standard Model and beyond, gravity antiscreens gauge couplings, generating an upper bound on the non-Abelian gauge coupling and allowing the non-Abelian gauge couplings to remain asymptotically free. The antiscreening nature also solves the Landau pole problem in the Abelian gauge sector and generates an upper bound on the fine-structure constant. The latter is compatible with the actually measured value.

- 在标准模型及超出标准模型的理论中，引力反屏蔽规范耦合，给非阿贝尔规范耦合生成了一个上界，并让非阿贝尔规范耦合能保持渐近自由。这种反屏蔽特性也解决了阿贝尔规范 sector 的朗道极点问题，给精细结构常数生成了一个上界，该上界与实际测量值相容。

- Gravity can either screen or antiscreen Yukawa couplings. The first possibility prohibits nonzero Yukawa couplings in the IR - if it is realized, asymptotically safe gravity is incompatible with the Standard Model. The second possibility gives rise to an upper bound on the Yukawa couplings. Results indicate that the second possibility may be realized, with the upper bound coinciding with the largest Yukawa coupling in the Standard Model, namely, the top Yukawa coupling, which thereby becomes calculable from first principles.

- 引力对汤川耦合既可以是屏蔽也可以是反屏蔽。第一种可能性会禁止红外区域存在非零汤川耦合——如果这种情况成立，渐近安全引力就与标准模型不相容。第二种可能性会给汤川耦合给出一个上界。研究结果表明第二种可能性可能成立，其上界恰好与标准模型中最大的汤川耦合 (即顶夸克汤川耦合) 一致，因此顶夸克汤川耦合可以从第一性原理计算得出。

- Gravity screens quartic couplings. In the Standard Model, this gives rise to a calculation of the ratio of the Higgs mass to the electroweak scale from first principles. Beyond the Standard Model, this drives portal couplings between the Standard Model and a dark scalar to zero, ruling out the simplest dark-matter model.

- 引力对四次耦合起到屏蔽作用。在标准模型中，这使得人们可以从第一性原理计算希格斯质量与电弱能标之比。超出标准模型的情况下，这种屏蔽作用会将标准模型与暗标量之间的门户耦合驱动到零，排除了最简单的暗物质模型。

- In extended dark sectors which contain scalars and gauge fields and/or fermions, portal couplings may be nonzero. Other couplings in these models are bounded from above by the asymptotic-safety requirement, ruling out part of the phenomenologically viable parameter spaces.

- 在包含标量、规范场和/或费米子的扩展暗 sector 中，门户耦合可以不为零。这些模型中的其他耦合受渐近安全要求的上限约束，排除了部分唯象上可行的参数空间。

- Axion-like particles decouple from photons, such that experimental searches for this popular dark-matter candidate are expected to yield null results in asymptotically safe gravity.

- 轴子样粒子与光子退耦，因此在渐近安全引力中，对这类热门暗物质候选者的实验搜索预期会得到零结果。

- Neutrino masses can be accommodated in asymptotically safe gravity-matter models through Yukawa couplings to the Higgs field. These Yukawa couplings are dynamically driven toward zero, giving rise to tiny neutrino masses. There is also an asymptotically safe fixed point at which a neutrino becomes exactly massless.

- 中微子质量可以通过与希格斯场的汤川耦合在渐近安全引力-物质模型中实现。这些汤川耦合会被动力学驱动趋近于零，从而产生极小的中微子质量。此外还存在一个渐近安全不动点，能让中微子恰好处于零质量状态。

- Physics beyond the Standard Model is constrained by the asymptotic-safety conjecture. Therefore, phenomenological models which are of interest to explain anomalies, i.e., tensions between Standard-Model predictions and experimental results, are not all on an equal footing, because only a subset of them is compatible with asymptotic safety.

- 超出标准模型的新物理受到渐近安全猜想的约束。因此，用于解释反常 (即标准模型预言与实验结果之间的偏差) 的唯象模型并非全部地位平等，因为只有其中一部分满足渐近安全的兼容性要求。

Outlook and Open Questions

展望与开放性问题

There are of course open questions in asymptotically safe settings with gravity and matter. First, there are several open questions that apply to asymptotically safe quantum gravity in general. These include, for example, unitarity, the Lorentzian signature, and convergence of results; see [77] for a thorough discussion. Second, there are open questions specifically within the interplay of gravity and matter. We will comment on some of them in the following.

当然，在引力与物质的渐近安全框架中仍存在开放性问题。首先，有若干开放性问题适用于一般的渐近安全量子引力，例如包括幺正性、洛伦兹号差以及结果的收敛性；全面讨论可参见文献 [77]。其次，在引力与物质的相互作用领域中存在专属于该方向的开放性问题。我们接下来将对其中部分问题进行探讨。

The Nature of Dark Matter

暗物质的本质

While dark-matter particles remain a compelling theoretical explanation of numerous astrophysical and cosmological observations, no such elementary particles have been detected yet. Therefore, there is a lot of freedom in constructing phenomenological models that can explain experimental observations. Typically, these models come with several free parameters and therefore have high-dimensional parameter spaces, which can only in part be probed experimentally. Embedding these dark-matter models within asymptotic safety can reduce the amount of free parameters significantly, as we have reviewed in this chapter. Furthermore, some models could even be ruled out by not admitting an asymptotically safe fixed point. Therefore, the principled-parametrized approach to dark matter in asymptotic safety may be a successful new paradigm for the nature of dark matter. It might educate model-building efforts and in the future even guide experimental searches for dark matter.

尽管暗物质粒子仍是对众多天体物理与宇宙学观测极具说服力的理论解释，目前仍未探测到这类基本粒子。因此，构建能够解释实验观测的唯象模型存在极大自由度。这类模型通常包含多个自由参数，因此参数空间维度很高，实验仅能对其进行部分探测。正如我们在本章中所回顾的，将这类暗物质模型嵌入渐近安全框架可以大幅减少自由参数的数量。此外，部分模型甚至会因不存在渐近安全不动点而被排除。因此，渐近安全框架下基于原则化参数化的暗物质研究方法，有望成为探索暗物质本质的成功新范式。它可以为暗物质的模型构建工作提供指导，未来甚至能引领暗物质的实验搜寻。

Explaining the Baryon Asymmetry of Our Universe

解释我们宇宙的重子不对称性

According to current models for the dynamics in the early universe, matter and antimatter are produced at the same rate. Furthermore, neither the SM nor general relativity includes a mechanism that generates a large enough asymmetry between baryons and anti-baryons. Thus, the origin of the observed asymmetry between the amount of matter and antimatter in the universe is still unknown. Just like for dark matter, there are phenomenological models which satisfy the Sakharov-conditions [216] and may thus explain the asymmetry. So far, within asymptotically safe quantum gravity, matter-antimatter asymmetry is mostly unexplored. Again, the principled-parametrized approach and the strong predictive power of scale symmetry might provide insights into which form of physics beyond the SM is viable.

根据当前早期宇宙动力学模型，物质与反物质的产生速率相同。此外，标准模型和广义相对论均不包含能产生足够大的重子与反重子不对称性的机制。因此，宇宙中观测到的物质与反物质总量不对称性的起源至今仍未可知。和暗物质的情况一样，现有一些满足萨哈罗夫条件 [216] 的唯象模型，或可解释该不对称性。到目前为止，渐近安全量子引力框架内对物质-反物质不对称性的研究几乎仍是空白。原理化参数化方法和标度对称性的强大预测能力，或能为我们探究超出标准模型的哪类物理是可行的提供新见解。

The Origin of Neutrino Masses

中微子质量的起源

To account for neutrino oscillations, neutrinos have to be massive. Various mechanisms for mass generation have been proposed and led to phenomenological models. Among this large number of models, some of which can be probed experimentally, the principled-parameterized approach is expected to select a smaller subset, providing us with theoretical guidance on the origin of neutrino masses.

为了解释中微子振荡现象，中微子必须具有质量。目前人们已提出多种质量产生机制，并建立了相应唯象模型。在这些大量可被实验探测的模型中，原理参数化方法有望从中筛选出更小的子集，为研究中微子质量的起源提供理论指导。

These three open questions - the nature of dark matter, the origin of matter-antimatter asymmetry, and the origin of neutrino masses - are just three examples of physics beyond the SM which is unexplained and described by a large number of phenomenological models, among which the principled-parameterized approach may select a smaller subset. More generally, we expect that the embedding into asymptotic safety and the description of this embedding through the principled-parameterized approach can be a powerful discriminator for model building. This can be useful in developing models to describe new physics and guide experiment. At the same time, it provides a way to derive potential phenomenological consequences of quantum gravity at energies much below the Planck scale. There is therefore the genuine possibility of ruling out asymptotically safe gravity-matter theories experimentally.

暗物质的本质、物质-反物质不对称的起源、中微子质量的起源这三个开放性问题，只是标准模型之外未得到解释的物理问题的三个例子，这些问题目前已有大量唯象模型，而原理参数化方法或许可以从中筛选出更小的子集。更广泛来说，我们认为将理论嵌入渐近安全范式、并通过原理参数化方法描述这一嵌入过程，可以成为模型构建的有力筛选工具。这对开发新物理描述模型、指导实验研究都很有帮助。同时，该方法还为推导出普朗克能标以下远很多的能区中量子引力可能的唯象效应提供了途径。因此，确实存在通过实验排除渐近安全引力-物质理论的可能性。

Cross-References

交叉引用

- Asymptotic Safety and Cosmology

- 渐近安全与宇宙学

- Asymptotic Safety of Gravity with Matter

- 含物质引力的渐近安全

Black Holes in Asymptotically Safe Gravity

渐近安全引力中的黑洞

Form Factors in Asymptotically Safe Quantum Gravity

渐近安全量子引力中的形状因子

Perturbative Approaches to Nonperturbative Quantum Gravity

非微扰量子引力的微扰方法

- Quantum Gravity and Scale Symmetry in Cosmology

- 宇宙学中的量子引力与标度对称性

- Quantum Gravity from Dynamical Metric Fluctuations

- 来自动力学度规涨落的量子引力

- The Functional f(R) Approximation

- 泛函 f(R) 近似

- The Functional Renormalization Group in Quantum Gravity

- 量子引力中的泛函重整化群

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